

Name: \_\_\_\_\_

Exam 1 (Version A)

**Instructions:** This exam is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Remove any smartwatches and non-religious head-wear. Cheating of any kind will not be tolerated and will result in a grade of zero. **You must clear the memory on your calculator before beginning the exam.** Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places. Write your answers on the test. You have **1 hour 15 minutes** to finish the exam. Answer all questions to the best of your ability. Unless otherwise specified, you are required to **SHOW ALL YOUR WORK** to receive full credit. The exam has 110 possible points. You will be graded out of 100 points.

Questions	Possible	Score		Possible	Score
Question 1	8		Question 8	8	
Question 2	8		Question 9	12	
Question 3	8		Question 10	16	
Question 4	8		Question 11	12	
Question 5	10		Extra Credit	3	
Question 6	9				
Question 7	8		Total	110	

1. For this problem you need only give your solution unless otherwise stated by the question. All questions are worth 1 point.

(a) Let  $f(x)$  be a function with respect to  $x$ . Then if  $f(x)$  gets smaller as  $x$  gets greater we say  $f(x)$  is \_\_\_\_\_.

(b) If  $H(n)$  is the height of a building in New York in feet and  $n$  is the number of windows, interpret  $H'(100) = 5$ .

(c) (T/F)  $e^{\ln(2x)} = x^2$ . \_\_\_\_\_

(d) (T/F) The average rate of change of a linear function is the slope. \_\_\_\_\_

(e) (T/F) If  $f'(a) > 0$  then  $f(x)$  is positive at  $a$ .

(f) (T/F) A company is making \$0 at the equilibrium quantity. \_\_\_\_\_

(g) (T/F) If an exponential function  $f(t)$  in terms of  $t$  has a constant percent increase  $r$  and initial value 500, then  $f(t) = 500r^t$ . \_\_\_\_\_

(h) If an objects weight,  $w$ , is inversely proportional to the cube of its distance,  $r$ , from earth's center, then what is a formula, given constant of proportionality  $k$ , for the relationship?

2. (8 points) Expand the following logarithm so that there no powers, no like terms, and no products/quotients.

$$\ln \left( \frac{5x^3y^3z^{-3}}{25x^3y^2z} \right)$$

3. (8 points) Let the surface area of the leaves of a tree,  $S$ , satisfy the function,  $S = kN^{1/5}$  where  $N$  is the number of leaves a tree has at a given time and  $k$  is the constant of proportionality. An oak tree has on average 3,200,000 leaves and a leaf surface area of 3000 inches squared.
- Find the constant of proportionality,  $k$  (**Show how you got your answer**).
  - Given the function with  $k$ , find the surface area when a tree has 24,300,000 leaves (**Show how you got your answer**).
4. (8 points) Aunt Marjie doesn't like going outside in the cold. Seriously...she's a big baby about it. If it's -10 F she'll only stay outside for 2 minutes, but if it's 70 F she'll stay out for 2 hours. Even at 70 though, she's still complaining.
- Find the average rate of change for time in minutes with respect to temperature in degrees Fahrenheit. Interpret the meaning of this in the context of the problem!
  - Find a linear function for the time,  $t$ , in minutes with respect to temperature,  $T$ , in degrees Fahrenheit.

5. (10 points) For each case below, draw an example of function which satisfies the conditions. Make sure to label  $a$  and  $b$  on the  $x$ -axis and  $f(a)$  and  $f(b)$  on the  $y$ -axis.

a.  $f(a) = f(b) > 0$  and  $f'(a) < f'(b)$  for  $a < b$ .

b.  $f''(a) < 0$ ,  $f''(b) = 0$ ,  $f'(c) = 0$ , and  $f(a) < 0$  for  $a < b < c$ .

6. (9 points) A company produces a good, incurring a fixed cost of \$1000. Producing each unit of the good costs the company an addition \$20. Suppose the company sells each unit of the good for \$45.

a. Find the cost, revenue and profit functions for the above company in terms of quantity,  $q$ .

b. Find the marginal cost, revenue and profit functions for the above company in terms of quantity,  $q$  and interpret what each means!

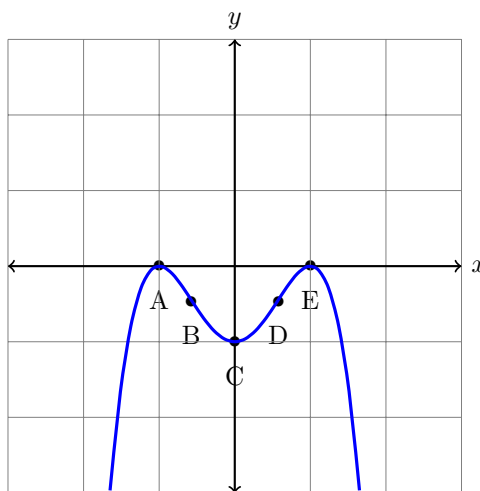
c. At what point,  $(q, \pi(q))$ , does the company break even?

7. (8 points) Given the graph below where  $A = (-1, 0)$ ,  $B = (-\sqrt{3}/3, -38/81)$ ,  $C = (0, -1)$ ,  $D = (\sqrt{3}/3, -38/81)$ , and  $E = (1, 0)$ , find the following:

a. Find the intervals for which the function is increasing and the intervals for which the function is decreasing and label which are increasing and which are decreasing (Note: Use open brackets for these intervals).

b. Find the intervals for which the function is concave up and the intervals for which the function is concave down and label which are concave up and which are concave down (Note: Use open brackets for these intervals).

c. Determine the inflection points! Please write them in the form  $(x, y)$ .



8. (8 points) Carrie, a marine biologist, is performing experiments along the continental slope off of the coast of Baja California where the biodiversity is very dynamic. Scientists have proven that biodiversity is closely linked to the temperature of the water. Carrie wants to monitor the temperature of the ocean at different depths along the continental slope, to help record the changes in biodiversity due to changes in the temperature of the water. At the same time she does not want the robot to crash into the continental slope, so she needs to take into effect the speed of the currents.

Earlier scientists have found that the speed of ocean current is a function of depth. The speed in meters per second,  $S$ , depends on depth in meters,  $d$ , according to the following formula.

$$S(d) = 3d + 1$$

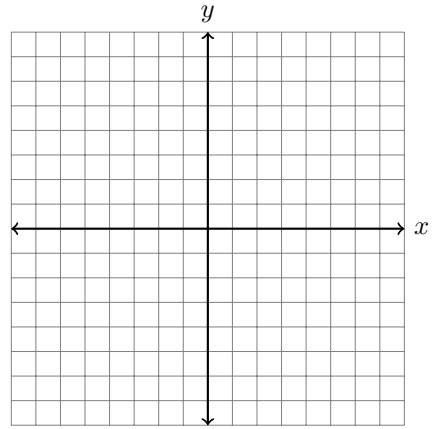
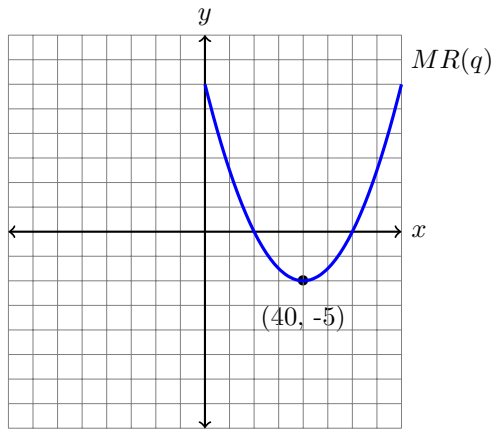
Where  $S$  is measured in meters per second and  $d$  is measured in meters. Suppose that the depth of a research robot depends on time in seconds,  $t$ , according to the formula:

$$d(t) = (1/9)(\pi t)^2$$

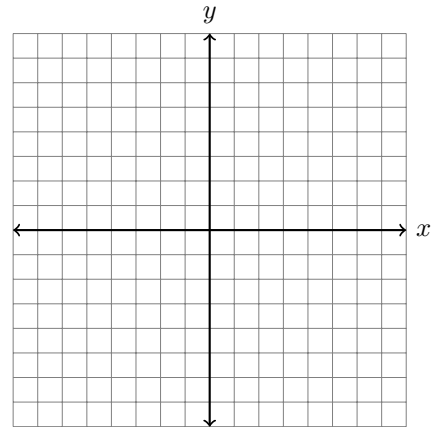
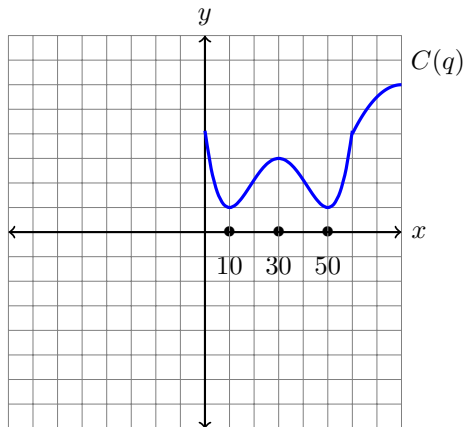
- a. Write a function that models the speed,  $S$ , of the current at the depth of the robot in terms of time,  $t$ .
- b. Determine if any of  $S(d)$ ,  $d(t)$ , and  $S(t)$  are power functions, linear functions, exponential, or none of the above (say which are). If one is a power function, give the constant of proportionality.

9. (12 points) Graphing!

- a. The graph of the marginal revenue curve is given below. Sketch the graph of the revenue curve.



- b. The graph of the cost curve is given below. Sketch the graph of the marginal cost curve.



10. (16 points) Let Matt Barnes invest \$2000 in an account compounded quarterly at a rate of 4% and let John Noble invest \$2000 in an account compounded continuously at a rate of 4%.
- Explain to Matt why John's account is better for earning money using the definition of compounding.
  - Find an function for the amount in the account,  $P$ , in terms of years,  $t$ , for each account.
  - Find the time at which the value of each account doubles (**Show how you got your answer**).



11. Let the demand curve be  $q = D(p) = -3p + 20$  and supply curve be  $q = S(p) = 7$ . Find the equilibrium  $(p^*, q^*)$  and graph the two curves. with  $p$  on the  $x$ -axis and  $q$  on the  $y$ -axis.

12. Extra Credit (3 points): Draw a picture of a dinosaur eating broccoli.