Math777: Graph Theory (II)
Spring, 2018
Homework 3, due Thursday, Mar. 8

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. [**page 289, #10**] Prove the following result of Schur: for every $k \in \mathbb{N}$ there is an $n \in \mathbb{N}$ such that, for every partition of $\{1, 2, \ldots, n\}$ into k sets, at least one of the subsets contains numbers x, y, z such that $x + y = z$.

2. [**page 289, #11**] A family of sets is called a Δ-system if every two of the sets have the same intersection. Show that every infinite family of sets of the same finite cardinality contains an infinite Δ-system.

3. [**page 290, #14**] Prove that $2^c < R(2, c, 3) \leq 3c!$ for every $c \in \mathbb{N}$.

4. [**page 290, #18**] Show that any Kuratowski set $\{\mathcal{P}_1, \ldots, \mathcal{P}_k\}$ of a given collection $\mathcal{C}$ of non-trivial graph properties is unique up to equivalence.

5. Let us 3-color the points of the plane. Prove that there will be two points at distance 1 with the same color.

6. Let us k-color all non-empty subsets of an n-element set. Prove that if n is large enough, there are two disjoint non-empty subsets X and Y such that X, Y, $X \cup Y$ have the same color.