# Math777: Graph Theory (II)
# Spring, 2018
# Homework 2, due Thursday, Feb. 15, 2018

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

**1.** **[page 195, #4** ] Determine the value of $ex(n, K_{1,r})$ for all $r, n \in \mathbb{N}$.

**2.** **[page 195, #5** ] Given $k > 0$, determine the extremal graphs without a matching of size $k$.

**3.** **[page 195, #9** ] Show that deleting at most $(m-s)(n-t)/s$ edges from a $K_{m,n}$ will never destroy all its $K_{s,t}$ subgraphs.

**4.** **[page 196, #11** ] Let $1 \leq r \leq n$ be integers. Let $G$ be a bipartite graph with bipartition $\{A, B\}$, where $|A| = |B| = n$, and assume that $K_{r,r} \not\subset G$. Show that

$$\sum_{x \in A} \binom{d(x)}{r} \leq (r-1)\binom{n}{r}.$$

Use it to deduce $ex(n, K_{r,r}) \leq cn^{2-1/r}$.

**5.** **[page 197, #20** ] Given a graph $G$ with $\epsilon(G) \geq k \in \mathbb{N}$, find a minor $H \prec G$ such that $\delta(H) \geq k \geq |H|/2$.

**6** If a graph $G_n$ contains no $K_4$ and only contains $o(n)$ independent vertices, then $\|G_n\| < (\frac{1}{8} + o(1))n^2$. (Hint: apply Szemerédi's Regularity Lemma.)