

Math777: Graph Theory (II)
Spring, 2018
Homework 1, due Thursday, Feb. 1

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. [page 165, #6] Let H be an abelian group, $G = (V, E)$ a connected graph, T a spanning tree, and f a map from the orientations of the edges in $E - E(T)$ to H that satisfies (F1). Show that f extends uniquely to a circulation on G with values in H .
2. [page 166, #15] Show that every graph with a Hamilton cycle has a 4-flow.
3. [page 166, #17] Determine the flow number of $C_5 * K_1$, the wheel with 5 spokes.
4. [page 166, #18] Find bridgeless graph G and $H = G - e$ such that $2 < \varphi(G) < \varphi(H)$.
5. [page 166, #20] Prove Heawood's theorem that a plane triangulation is 3-colorable if and only if all its vertices have even degree.
6. [page 167, #23] Show that a graph $G = (V, E)$ has a k -flow if and only if it admits an orientation D that directs, for every $X \subset V$, at least $\frac{1}{k}$ of the edges in $E(X, \bar{X})$ from X towards \bar{X} .