Math776: Graph Theory (I) Fall, 2017 Homework 5, due Monday, Dec. 4

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

- 1. [page 111, #4] show that every planar graph is a union of three forests.
- 2. [page 112, #20] Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a TK_5 and a $TK_{3,3}$ subgraph.
- **3.** [page 112, #22] A graph is called *outplanar* if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither K_4 nor $K_{2,3}$ as a minor.
- 4. [page 140, #13] Show that every critical k-chromatic graph is (k 1)-edge-connected.
- 5. [page 140, #24] For every k, find a 2-chromatic graph whose choice number is at least k.
- 6. [page 140, #13] Prove that the choice number of K_2^r is r. (Here K_2^r is the complete r-partite graph with each part of size 2.)