# Math776: Graph Theory (I) <br> Fall, 2017 <br> Homework 5, due Monday, Dec. 4 

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. [page 111, \#4 ] show that every planar graph is a union of three forests.
2. [page 112, \#20 ] Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a $T K_{5}$ and a $T K_{3,3}$ subgraph.
3. [page 112, \#22] A graph is called outplanar if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither $K_{4}$ nor $K_{2,3}$ as a minor.
4. [page 140, \#13 ] Show that every critical $k$-chromatic graph is $(k-1)$ -edge-connected.
5. [page 140, \#24 ] For every $k$, find a 2-chromatic graph whose choice number is at least $k$.
6. [page $140, \# 13$ ] Prove that the choice number of $K_{2}^{r}$ is $r$. (Here $K_{2}^{r}$ is the complete $r$-partite graph with each part of size 2 .)
