Math776: Graph Theory (I)
Fall, 2017
Homework 4, due Wednesday, Nov. 8

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. [**page 83, #4**] Let X and X' be minimal separators in G such that X meets at least two components of $G - X'$. Show that X' meets at least two components of $G - X$, and X meets all the components of $G - X'$.

2. [**page 83, #10**] Let e be an edge in a 3-connected graph $G \neq K_4$. Show that either $G \div e$ or G/e is again 3-connected.

3. [**page 84, #18**] Let $k \geq 2$. Show that every k-connected graph of order at least $2k$ contains a cycle of length at least $2k$.

4. [**page 84, #19**] Let $k \geq 2$. Show that in a k-connected graph any k vertices lie on a common cycle.

5. [**page 84, #24**] Derive Tutte's 1-factor theorem from Mader's theorem.

6. [**page 84, #26**] For every $k \in \mathbb{N}$ find an $l = l(k)$, as large as possible, such that not every l-connected graph is k-linked.