

Math776: Graph Theory (I)
Fall, 2017
Homework 3, due Wednesday, Oct. 18

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. [page 31, #39] Prove Gallai's theorem that the edge set of any graph G can be written as a disjoint union $E(G) = C \cup D$ with $C \in \mathcal{C}(G)$ and $D \in \mathcal{C}^*(G)$.
2. [page 54, #11] Let G be a bipartite graph with bipartition $\{A, B\}$. Assume that $\delta(G) \geq 1$, and that $d(a) \geq d(b)$ for every edge ab with $a \in A$. Show that G contains a matching of A .
3. [page 55, #5] Derive the marriage theorem from König's theorem.
4. [page 55, #8] Find an infinite counterexample to the statement of the marriage theorem.
5. [page 55, #9] Let A be a finite set with subsets A_1, \dots, A_n , and let $d_1, \dots, d_n \in \mathbb{N}$. Show that there are disjoint subsets $D_k \subset A_k$, with $|D_k| = d_k$ for all $k \leq n$ if and only if

$$|\cup_{i \in I} A_i| \geq \sum_{i \in I} d_i$$

for all $I \subset \{1, \dots, n\}$.

6. [page 55, #14] Show that all stable matchings of a given graph cover the same vertices. (In particular, they have the same size.)