## Math776: Graph Theory (I) Fall, 2017 Homework 2, due Wednesday, Oct. 4

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

- 1. [page 31, #20] Show that a graph is 2-edge-connected if and only if it has a *strongly-connected* orientation, one in which every vertex can be reached from every other vertex by a directed path.
- 2. [page 31, #21 ] Find a short inductive proof for the existence of normal spanning trees in finite connected graphs.
- 3. [page 31, #24 ] Show that every automorphism of a tree fixes a vertex or an edge.
- 4. [page 32, #27] Prove or disprove that a graph is bipartite if and only if no two adjacent vertices have the same distance from any other vertices.
- 5. [page 32, #28] Find a function  $f : \mathbb{N} \to \mathbb{N}$  such that, for all  $k \in \mathbb{N}$ , every graph of average degree at least f(k) has a bipartite subgraph of minimum degree at least k.
- 6. [page 32, #30 ] Prove or Disprove that every connected graph contains a walk that traverses each of its edges exactly once in each direction.