Math776: Graph Theory (I)
Fall, 2017
Homework 1, due Monday, Sept. 11

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly.

1. **[page 30, #2**] Determine the average degree, number of edges, diameter, girth, and circumference of the hypercube graph Q_d.

2. **[page 30, #3**] Let G be a graph containing a cycle C, and assume that G contains a path of length at least k between two vertices of C. Show that G contains a cycle of length at least $\sqrt{k}$.

3. **[page 30, #8**] Show that every connected graph G contains a path of length at least $\min\{2\delta(G), |G| - 1\}$.

4. **[page 30, #9**] Show that a connected graph of diameter k and minimum degree d has at least about $kd/3$ vertices but need not have substantially more.

5. **[page 30, #12**] Determine $\kappa(G)$ and $\lambda(G)$ for $G = P_m, C_n, K_n, K_{m,n}$, and Q_d; $d, m, n \geq 3$.

6. **[page 31, #18**] Show that a tree without a vertex of degree 2 has more leaves than other vertices. Can you find a very short proof that does not use induction?