Math777: Graph Theory (II)
Spring, 2014
Homework 2, due Tuesday, Mar. 4

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. [**page 195, #4**] Determine the value of $ex(n, K_{1,r})$ for all $r, n \in \mathbb{N}$.

2. [**page 195, #5**] Given $k > 0$, determine the extremal graphs without a matching of size k.

3. [**page 196, #11**] Let $1 \leq r \leq n$ be integers. Let G be a bipartite graph with bipartition $\{A, B\}$, where $|A| = |B| = n$, and assume that $K_{r,r} \not\subseteq G$. Show that

$$\sum_{x \in A} \binom{d(x)}{r} \leq (r-1)\binom{n}{r}.$$

Use it to deduce $ex(n, K_{r,r}) \leq cn^{2-1/r}$.

4. [**page 196, #13**] Given a tree T, find an upper bound for $ex(n, T)$ that is linear in n and independent of the structure of T, i.e., depends only on $|T|$.

5. [**page 197, #20**] Given a graph G with $\epsilon(G) \geq k \in \mathbb{N}$, find a minor $H \prec G$ such that $\delta(H) \geq k \geq |H|/2$.

6 If a graph G_n contains no K_4 and only contains $o(n)$ independent vertices, then $\|G_n\| < (\frac{1}{8} + o(1))n^2$. (Hint: apply Szemerédi's Regularity Lemma.)