Math777: Graph Theory (II)
Spring, 2014
Homework 1, due Thursday, Jan. 30

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. **[page 165, #3**] Derive Menger's Theorem 3.3.5 from the max-flow min-cut theorem.

2. **[page 166, #14**] Let G be a bridgeless connected graph with n vertices and m edges. By considering a normal spanning tree of G, show that $\varphi(G) \leq m - n + 2$.

3. **[page 166, #15**] Show that every graph with a Hamilton cycle has a 4-flow.

4. **[page 166, #18**] Find bridgeless graph G and $H = G - e$ such that $2 < \varphi(G) < \varphi(H)$.

5. **[page 166, #20**] Prove Heawood's theorem that a plane triangulation is 3-colorable if and only if all its vertices have even degree.

6. **[page 167, #23**] Show that a graph $G = (V, E)$ has a k-flow if and only if it admits an orientation D that directs, for every $X \subset V$, at least $\frac{1}{k}$ of the edges in $E(X, \bar{X})$ from X towards $\bar{X}$.