## MATH778 Large Networks and Graph Limits Homework 3, due Dec. 4

1. Prove that the number of perfect matchings in a graph $G=(V, E)$ can be expressed as $t\left(G, e^{-2 \pi i x}, 1+e^{2 \pi i(x+y)}\right)$.
2. Suppose that two kernels $U$ and $W$ are weakly isomorphic. Prove that so are the kernels $a U+b$ and $a W+b(a, b \in \mathbb{R})$.
3. Let $W$ be a graphon. Prove that
(a) all eigenvalues of $T_{W}$ are contained in the interval $[-1,1]$;
(b) the largest eigenvalue is also largest in absolute value;
(c) at least one of the eigenvectors belonging to the largest eigenvalue is nonnegative almost everywhere.
4. Let $A$ be a symmetric $n \times n$ matrix with all entries in $[-1,1]$. Let $A^{\prime}$ be obtained from A by deleting a row and the corresponding column. Prove that

$$
\left|\|A\|_{\square}-\left\|A^{\prime}\right\|_{\square}\right| \leq \frac{2}{n}
$$

5. Prove that for any stepfunction $U$ with $k$ steps,

$$
\|U\|_{1} \leq 2 k\|U\|_{\square} .
$$

6. Show that $\left\|W_{n}\right\|_{\square} \rightarrow 0\left(W_{n} \in \mathcal{W}_{1}\right)$ does not imply that $\left\|W_{n}\right\|_{1} \rightarrow 1$.
