MATH778 Large Networks and Graph Limits Homework 3, due Dec. 4

- 1. Prove that the number of perfect matchings in a graph G = (V, E) can be expressed as $t(G, e^{-2\pi i x}, 1 + e^{2\pi i (x+y)})$.
- 2. Suppose that two kernels U and W are weakly isomorphic. Prove that so are the kernels aU + b and aW + b $(a, b \in \mathbb{R})$.
- 3. Let W be a graphon. Prove that
 - (a) all eigenvalues of T_W are contained in the interval [-1, 1];
 - (b) the largest eigenvalue is also largest in absolute value;
 - (c) at least one of the eigenvectors belonging to the largest eigenvalue is nonnegative almost everywhere.
- 4. Let A be a symmetric $n \times n$ matrix with all entries in [-1, 1]. Let A' be obtained from A by deleting a row and the corresponding column. Prove that

$$|||A||_{\Box} - ||A'||_{\Box}| \le \frac{2}{n}.$$

5. Prove that for any stepfunction U with k steps,

$$||U||_1 \le 2k ||U||_{\square}.$$

6. Show that $||W_n||_{\Box} \to 0$ $(W_n \in \mathcal{W}_1)$ does not imply that $||W_n||_1 \to 1$.