MATH778 Large Networks and Graph Limits
Homework 2, due Nov. 13

1. (a) Prove that the strong product of two graphs is connected if and only if both graphs are connected. (b) Show by an example that the categorical product of two connected graphs is not always connected. Characterize all counterexamples in (b).

2. Prove that if all nodes of a simple graph F are labeled, then both F and the quantum graph $\hat{F}$ are idempotent in the algebra of simple partially labeled graphs.

3. Let z be a contractor for a graph parameter f, and let F be a k-labeled graph. Let us delete an edge $1i$ from F, and add the edge $2i$, to obtain another k-labeled graph F'. Prove that $zF \cong zF' \pmod{f}$.

4. Prove that the number of eurlerian orientation $\overrightarrow{eul}$ is contractible, but has no contractor.

5. Prove that for every weighted graph H with q nodes and every $t \geq 2$, $\hom(\bullet, H)$ has a connector whose constituents are $P_t^{\bullet,\bullet}$, $P_{t+1}^{\bullet,\bullet}$, ..., $P_{t+q}^{\bullet,\bullet}$.

6. Show by an example that $\hom(G, Z)$ can be real for every multi-graph G for a non-real matrix Z.