MATH778 Large Networks and Graph Limits
Homework 1, due Sept. 30

1. For K_3-free graphs, prove $\leq \frac{1}{2}$ and $\leq \frac{1}{4}$.

2. Prove $\geq$.

3. For any $a, b \geq 0$ and any simple graph G, prove that $t(P_{2a+2b+1}, G)^2 \leq t(P_{2a+1}, G)t(P_{2b+1}, G)$.

4. Prove that for every isolate-indifferent graph parameter f, the connection rank $r(f, k)$ is a monotone non-decreasing function of k.

5. Prove that the graph parameter $2^{\alpha(G)}$ is multiplicative and have finite connection rank.

6. Show that every minor-monotone graph property can be expressed by a node-monadic second-order formula.