Math776: Graph Theory (I)
Fall, 2013
Homework 6, due Friday, Dec. 13

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. [**page 112, #20**] Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a TK_5 and a $TK_{3,3}$ subgraph.

2. [**page 112, #22**] A graph is called *outplanar* if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither K_4 nor $K_{2,3}$ as a minor.

3. [**page 114, #37**] Let G, G^* be dual plan graphs. Prove the following statements:

1. If G is 2-connected, then G^* is 2-connected.
2. If G is 3-connected, then G^* is 3-connected.
3. If G is 4-connected, then G^* need not be 4-connected.

4. [**page 140, #13**] Show that every critical k-chromatic graph is $(k-1)$-edge-connected.

5. [**page 140, #24**] For every k, find a 2-chromatic graph whose choice number is at least k.

6. [**page 140, #13**] Prove that the choice number of K_2^r is r. (Here K_2^r is the complete r-partite graph with each part of size 2.)