Math776: Graph Theory (I)
Fall, 2013
Homework 4, due Monday, Oct. 21

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. [**page 54, #11**] Let G be a bipartite graph with bipartition $\{A, B\}$. Assume that $\delta(G) \geq 1$, and that $d(a) \geq d(b)$ for every edge ab with $a \in A$. Show that G contains a matching of A.

2. [**page 55, #14**] Show that all stable matchings of a given graph cover the same vertices. (In particular, they have the same size.)

3. [**page 55, #15**] Show that the following 'obvious' algorithm need not produce a stable matching in a bipartite graph. Starting with any matching. If the current matching is not maximal, add an edge. If it is maximal but not stable, insert an edge that creates instability, deleting any current matching edges at its ends.

4. [**page 55, #20**] Derive the marriage theorem form Tutte's theorem.

5. [**page 83, #4**] Let X and X' be minimal separators in G such that X meets at least two components of $G - X'$. Show that X' meets at least two components of $G - X$, and X meets all the components of $G - X'$.

6. [**page 83, #10**] Let e be an edge in a 3-connected graph $G \neq K_4$. Show that either $G \dot{-} e$ or G/e is again 3-connected.