# Math776: Graph Theory (I)
# Fall, 2013
# Homework 3, due Friday, Oct. 4

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

**1.** A $k$-ary deBruijn sequence $B(k, s)$ of order $s$ is a cyclic sequence of a given alphabet $A$ with size $k$ for which every possible subsequence of length $s$ in $A$ appears as a sequence of consecutive characters exactly once. The deBruijn sequences can be constructed by taking an Eulerian tour of a directed graph $D = (V, E)$, where $V$ consists of all strings of length $s - 1$ of characters in $A$ and $E$ consists of all pairs of strings so that the second string can be obtained from the first one by removing the first character and appending another one at the end. Show that any Eulerian Tour of $D$ gives a deBruijn sequence. Construct a deBruijn sequence $B(2, 4)$ using this approach.

**2.** [**page 32, #32** ] Show that the cycle space of a graph is spanned by

- its induced cycles;
- its geodetic cycles.

(A cycle $C \subset G$ is *geodetic* in $G$ if, for every two vertices of $C$, their distances in $G$ equals their distance in $C$.)

**3.** [**page 31, #39** ] Prove Gallai's theorem that the edge set of any graph $G$ can be written as a disjoint union $E(G) = C \cup D$ with $C \in \mathcal{C}(G)$ and $D \in \mathcal{C}^*(G)$.

**4.** [**page 55, #5** ] Derive the marriage theorem from König's theorem.

**5.** [**page 55, #8** Find an infinite counterexample to the statement of the marriage theorem.

**6.** [**page 55, #9** ] Let $A$ be a finite set with subsets $A_1, \ldots, A_n$, and let $d_1, \ldots, d_n \in \mathbb{N}$. Show that there are disjoint subsets $D_k \subset A_k$, with $|D_k| = d_k$ for all $k \leq n$ if and only if

$$|\cup_{i \in I} A_i| \geq \sum_{i \in I} d_i$$

for all $I \subset \{1, \ldots, n\}$.