Math776: Graph Theory (I)
Fall, 2013
Homework 2, due Friday, Sept. 20

Select any 5 problems to solve. The total score of this homework is 10 points. You get a bonus point if you solve all 6 problems correctly. You also get another bonus point if your solution is selected as a standard solution (in this case you will be asked to send me the latex code of this solution.)

1. [**page 31, #20**] Show that a graph is 2-edge-connected if and only if it has a *strongly-connected* orientation, one in which every vertex can be reached from every other vertex by a directed path.

2. [**page 31, #21**] Find a short inductive proof for the existence of normal spanning trees in finite connected graphs.

3. [**page 31, #24**] Show that every automorphism of a tree fixes a vertex or an edge.

4. [**page 32, #27**] Prove or disprove that a graph is bipartite if and only if no two adjacent vertices have the same distance from any other vertices.

5. [**page 32, #28**] Find a function $f : \mathbb{N} \to \mathbb{N}$ such that, for all $k \in \mathbb{N}$, every graph of average degree at least $f(k)$ has a bipartite subgraph of minimum degree at least k.

6. [**page 32, #30**] Prove or Disprove that every connected graph contains a walk that traverses each of its edges exactly once in each direction.