

The Research Statement of Linyuan Lu

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Dr. Lu has been working on various extremal, probabilistic, and spectral problems in combinatorics. He has published one book, one book chapter, 33 Journal papers, and 9 conference papers. He also has 6 submitted papers currently under review.

Dr. Lu's research lies the following areas: large information networks, sparse random graphs, spectral graph theory, probabilistic methods, spectral graph/hypergraph theory, Ramsey type problems, algorithms, and other problems in graph theory. The results in four areas are presented first; other significant results for the selected papers (not in the areas) are listed at the end.

Large information networks:

Dr. Lu made major contributions to the theory of large sparse random graphs — a new direction in graph theory for modeling many real-world graphs. Together with Chung (and other collaborators), Lu introduced and studied several random graph models, including models using preferential attachment scheme, partial duplication models, random graphs with expected degree sequence, and hybrid models. They derived many graph properties, such as the giant connected component, the average distance, the diameter, and spectra.

Lu and Chung [CL06a] published a book “Complex graphs and networks” (264 pages, AMS publisher) in 2006. This book summarizes their major contribution to random graph theory in the area of large sparse graphs: a dozen of papers written by Lu, Chung, and their coauthors [ACL00, ACL01b, CL01, CL04b, ACL02, ACL01a, CL02b, CL06c, CL02a, CLV03b, CLV03a, CLDG03, ACL04, WLC03, CL06b, ACL05]. Graph theory has emerged as a primary tool for detecting numerous hidden structures in various information networks including Internet graphs, social networks, biological networks, or, more generally, any graph representing relations in massive data sets. These real-world graphs are very large, sparse, and often having the power law degree distributions. In the power law degree distribution, the number of vertices with degree k is proportional to $k^{-\beta}$. Here β is the exponent of power law graphs. In their early work [ACL00], Aiello, Chung, and Lu first applied random graphs with given degree sequence to study the connected components of power law graphs. Aiello, Chung, and Lu [ACL01b, ACL02] suggested several evolution models of preferential attachment scheme and proved the exponent β is an invariant for power law graph models under different time scale. They gave a rigorous explanation of “scale-freeness”, which has been observed by many authors. The models using preferential attachment scheme can only generate the power law graphs with exponent $\beta > 2$. The power law graphs in this range includes many non-biological networks such as the Internet graphs, the collaboration graph, and social networks. However, biological networks, such as gene regulatory graphs, protein-protein graphs, and ecological networks, are power law graphs with $\beta < 2$. Chung, Lu, Dewey, and Galas [CLDG03] introduced the partial duplication models, which are among a

few models that can generate power law graphs with $\beta < 2$.

The models using the preferential attachment scheme are good at generating power law degree distributions. However, they are usually hard to analyze. Chung and Lu [CL04a] showed that the preferential attachment model with vertex-deletion could be coupled with the random graph model with given expected degree sequences. Thus, many results of $G(w_1, \dots, w_n)$ (defined later) can be carried out to the preferential attachment model(s).

The well-observed small world phenomenon includes both small average distance and the clustering effect, which is not possessed by random graphs. Chung and Lu [ACL04] use a hybrid model which combines a global graph (a random power law graph) with a local graph (a graph with high local connectivity defined by network flow). They present an efficient algorithm, called PARTITION, which extracts a local graph from a given realistic network. They showed that the hybrid model was robust in the sense that for any graph generated by the hybrid model, the PARTITION algorithm approximately recovers the local graph. The PARTITION algorithm results in a new way to draw real-world graphs [ACL07].

Sparse random graphs:

Chung and Lu [CL02b] introduced the model $G(w_1, \dots, w_n)$ for random graph with given degree sequences. For each pair (i, j) , the edge $v_i v_j$ is added independently with probability $\frac{w_i w_j}{\sum_{i=1}^n w_i}$. Erdős-Rényi's $G(n, p)$ model is a special case of this model with equal weights. Let $d = \frac{\sum_{i=1}^n w_i}{n}$ and $\tilde{d} = \frac{\sum_{i=1}^n w_i^2}{\sum_{i=1}^n w_i}$. For any set S , the volume of S is defined to be $\text{Vol}(S) = \sum_{v_i \in S} w_i$. If $\tilde{d} < 1$, it is shown in [CL02b] that almost surely all components have volume at most $O(\sqrt{n} \log n)$. If $d > 1$, Chung and Lu [CL06c] proved almost surely there is a unique giant connected component (GCC) with volume $(\lambda_0 + O(\sqrt{n} \frac{\log^{3.5} n}{\text{Vol}(G)})) \text{Vol}(G)$. Here λ_0 is the unique positive root of the following equation: $\sum_{i=1}^n w_i e^{-w_i \lambda} = (1 - \lambda) \sum_{i=1}^n w_i$. Chung and Lu [CL02a] proved almost surely the average distance of $G(w_1, \dots, w_n)$ is $(1 + o(1)) \frac{\log n}{\log d}$ if weights $w_1, \dots, w_n$ are admissible. The diameter is almost surely $\Theta(\frac{\log n}{\log d})$ under strongly admissible condition. These results hold for random power law graphs with $\beta > 3$. However, for $\beta = 3$, almost surely the average distance is $O(\frac{\log n}{\log \log n})$ while the diameter is still $\Theta(\log n)$. For $2 < \beta < 3$, almost surely the average distance is $O(\log \log n)$ while the diameter is still $\Theta(\log n)$. Random power law graphs have an interesting “octopus” shape. It has a dense core of diameter $O(\log \log n)$ and a few long legs of length $\Theta(\log n)$. Chung, Lu, and Van [CLV03b] proved that the Laplacian eigenvalues of $G(w_1, \dots, w_n)$ follows the semi-circle law with spectral radius at most $(1 + o(1)) \frac{4}{\sqrt{d}}$ if $w_{\min} \gg \sqrt{d} \log^2 n$. For the eigenvalues of adjacency matrix, Chung, Lu, and Van [CLV03a] proved that the largest eigenvalue μ_1 is essentially the maximum of $\sqrt{w_{\max}}$ and $\tilde{d}$, if they are apart by at least a factor of $\log^2 n$. If $\tilde{d}$ is small enough ($< \frac{\sqrt{w_k}}{\log^2 n}$), the k -th largest eigenvalue is about the square root of k -th largest weight w_k . It implies for random power law graphs with $\beta > 2.5$ the eigenvalues of adjacency matrix have a triangular-like distributions, which was observed by [FFF99, FDBV01, MP02].

Probabilistic methods:

Dr. Lu [CL06b, LS07, LS] also made contributions to fundamental tools of probabilistic methods. In a survey paper [CL06b], Chung and Lu extended Chernoff's inequality and Azumar's martingale inequality so that they can be used in general random graphs with uneven degree distribution. These inequalities have been found and included in the papers [CL02b, CL02a, CLV03b, CL04a]. The survey paper [CL06b] puts together these extensions and generalizations to present a more complete picture. It should be useful for other scenarios.

Drs. Lu and Székely [LS07, LS] discovered an easy way to apply the Lovász Local Lemma to the probability space of random matchings in the complete graph K_N , or the complete bipartite graph $K_{N,M}$. They proved a sufficient condition of "negative dependency graph" among the events in these probability spaces of matchings. They have already found many interesting applications such as asymptotic permutation enumeration, packing problem of hypergraphs, and Turán type extremal problems. For example, here is an interesting result on Turán number of hypergraphs [LS07]. For a fixed r -graph G , let $\pi(G)$ be the Turán density of G . Suppose every edge of G intersects at most d other edges, then $\pi(G) \leq 1 - \frac{1}{(d+1)e}$. It is interesting that a local parameter d can control the Turán density $\pi(G)$. In the second paper [LS], they also proved tight upper bounds that asymptotically match the lower bound given by the Lovász Local Lemma. As a consequence, they gave new proofs to a number of results on the enumeration of permutations, Latin rectangles, and regular graphs. The strength of the method is shown by a new result: enumeration of graphs by degree sequence or bipartite degree sequence and girth.

Extremal problems on hypergraphs and posets:

Dr. Lu is also interested in the Turán number of hypergraphs. Given a family of r -uniform hypergraphs $\mathcal{H}$, the Turán density $\pi(\mathcal{H})$ is the least number α so that any r -uniform hypergraph on n -vertices with edge density $\alpha + o_n(1)$ must contain an induced subgraph in $\mathcal{H}$. Let K_k^r denote the complete r -graph on k vertices. No Turán density $\pi(K_k^r)$ is known for any $k > r \geq 3$. Erdős [Erd81] offered \$500 for determining any $\pi(K_k^r)$ with $k > r \geq 3$ and \$1000 for settling the whole problem. Chung and Lu [CL99] proved $\pi(K_4^3) \leq \frac{\sqrt{21}-1}{6} = .5971\dots$. Let G_1 be the 3-graph on four vertices with one edge. Razborov [Raz10] proved $\pi(\{K_4^3, G_1\}) = \frac{5}{9}$ (and $\pi(K_4^3) \leq 0.56\dots$) using flag algebra method. (Now Razborov's flag algebra method becomes the de facto method to attack various Turán type problems for small configurations.)

For odd $r \geq 3$, Chung and Lu [CL99] also proved $\pi(K_{r+1}^r) \leq 1 - \frac{5r+12-\sqrt{9r^2+24r}}{2r(r+3)}$. For even $r \geq 4$, Lu and Zhao [LZ09] proved that $\pi(K_{r+1}^r) \leq 1 - \frac{1}{r} - (1 - \frac{1}{r^{p-1}}) \frac{(r-1)^2}{2r^p((\frac{r+p}{p-1}) + (\frac{r+1}{2}))}$. Here p is the smallest prime factor of $r-1$. Lu and Zhao also proved an exact result for hypergraphs. If $n > (p-1)r$ and G is an r -uniform hypergraph on $[n]$ such that every $r+1$ vertices contain 0 or r edges, then $E(G)$ is either empty or $\{E \subset [n] : |E| = r, E \ni x\}$ for some $x \in [n]$.

Lu and Griggs [Gri09, GLL12] works on extremal set problems. Given a finite poset P , we consider the largest size $\text{La}(n, P)$ of a family of subsets of $[n] := \{1, \dots, n\}$ that contains no (weak) subposet P . This problem has been studied intensively in recent years, and it is conjectured that $\pi(P) := \lim_{n \rightarrow \infty} \text{La}(n, P) / \binom{n}{\lfloor \frac{n}{2} \rfloor}$ exists for general posets P , and, moreover, it is an integer. Katona and others [DBK07, DBKJ05, GK08, Tha98] have investigated the behavior of $\text{La}(n, H)$, which denotes the maximum size of H -free families $\mathcal{F} \subset 2^{[n]}$. Lu and Griggs [Gri09] use a new approach, which is to apply methods from extremal graph theory and probability theory to identify new classes of posets H , for which $\text{La}(n, H)$ can be determined asymptotically as $n \rightarrow \infty$ for various posets H , including two-end-forks, up-down trees, and cycles C_{4k} on two levels. Their result can be stated as follows. If H is a two-end-forks with height h ($h \geq 3$), then $\pi(H) = h - 1$. If H is an up-down tree, then $\pi(H) = 1$. If H is a cycle C_{4k} on two levels, then $\pi(C_{4k}) = 1$ (for $k \geq 2$). For $k \geq 2$ let $\mathcal{D}_k$ denote the k -diamond poset $\{A < B_1, \dots, B_k < C\}$. Griggs, Lu, and Li [GLL12] study the average number of times a random full chain meets a P -free family, called the Lubell function, and use it for $P = \mathcal{D}_k$ to determine $\pi(\mathcal{D}_k)$ for infinitely many values k . A stubborn open problem is to show that $\pi(\mathcal{D}_2) = 2$; they make progress by proving $\pi(\mathcal{D}_2) \leq 2 \frac{3}{11}$ (if it exists).

Other significant results from selected papers not in 4 areas above:

Paper 2: Linyuan Lu and Xing Peng, On Meyniel's conjecture of the cop number, *Journal of Graph Theory*, accepted.

Given a graph G , the cop number $c(G)$ is the minimum integer c so that c cops can capture the robber on G in the cop-robbers game introduced by Nowakowski and Winkler [NW83], and by Quilliot [Qui83]. Meyniel conjectured that the cop number $c(G)$ of any connected graph G on n vertices is at most $C\sqrt{n}$ for some constant C . Lu and Peng [LP11b] prove Meyniel's conjecture in special cases that G has diameter 2 or G is a bipartite graph of diameter 3. For general connected graphs, we prove $c(G) = O(n2^{-(1-o(1))\sqrt{\log_2 n}})$, improving the best previously known upper-bound $O(\frac{n}{\ln n})$ due to Chiniforooshan. The same upper bound is proved independently by Sudakov and Scott [SB11]. Frieze, Krivelevich, and Loh [FKP11] generalized this result to a variation game where the robber can move R edges at a time.

Paper 5: Linyuan Lu and Yiting Yang, The Randic index and the diameter of graphs, *Discrete Mathematics*, **311**, Issue 14, (2011) 1333-1343.

The *Randić index* $R(G)$ of a graph G is defined as the sum of $\frac{1}{\sqrt{d_u d_v}}$ over all edges uv of G , where d_u and d_v are the degrees of vertices u and v , respectively (see [M.75, BE98].) Let $D(G)$ be the diameter of G when G is connected. Aouchiche-Hansen-Zheng (2007) [AHZ07] conjectured that among all connected graphs G on n vertices the path P_n achieves the minimum values for both $R(G)/D(G)$ and $R(G) - D(G)$. Yang and Lu [Yan11] proved this conjecture completely. In fact, they proved a stronger theorem: If G is a connected graph, then $R(G) - \frac{1}{2}D(G) \geq \sqrt{2} - 1$, with equality if and only if G is a path with at least

three vertices.

Paper 8: Wei-Tian Li, Linyuan Lu, and Yiting Yang, Routing numbers of Cycles, Complete Bipartite Graphs, and Hypercubes, *SIAM J. Discrete Math.* **24**, (2010), 1482-1494.

In this paper, Li, Lu, and Yang study the routing numbers of cycles, complete bipartite graphs, and hypercubes. The routing number $rt(G)$ of a connected graph G is the minimum integer r so that every permutation of vertices can be routed in r steps by swapping the ends of disjoint edges. They [LLY10] proved that $rt(C_n) = n - 1$ (for $n \geq 3$) and for $s \geq t$, $rt(K_{s,t}) = \lfloor \frac{3s}{2t} \rfloor + O(1)$. They also proved $n+1 \leq rt(Q_n) \leq 2n-2$ for $n \geq 3$. The lower bound $rt(Q_n) \geq n+1$ was previously conjectured by Alon, Chung and Graham [ACG94]. Li, Lu, and Yang [LLY10] generalized the routing number $rt(G)$ to the fractional routing number $rt'(G)$. They proved $rt'(P_n) = rt'(S_n) = n$ for $n \geq 3$, and conjectured $rt'(T) = n + O(1)$ for any tree T .

Paper 9: Linyuan Lu and Yiting Yang, A Lower bound of transposition diameter for permutations, *SIAM J. Discrete Math.* **24** (2010) 1242-1249.

Sorting permutations by transpositions is an important and difficult problem in genome rearrangements. The transposition diameter $TD(n)$ is the maximum transposition distance among all pairs of permutations in S_n . It was previously conjectured [EEK⁺01] that $TD(n) \leq \lceil \frac{n+1}{2} \rceil$. This conjecture was disproved by Elias and Hartman [ET06] by showing $TD(n) \geq \lfloor \frac{n+1}{2} \rfloor + 1$. In this paper, Lu and Yang improved the lower bound to $TD(n) \geq \frac{17}{33}n + \frac{1}{33}$, via computation.

Paper 13: Linyuan Lu, Explicit construction of small Folkman graphs, *SIAM J. Discrete Math.*, **21** (2008), No. 4, 1053-1060.

In this paper, Lu solved an Erdős \$100 problem on Folkman Graphs. In 1967 Erdős and Hajnal [EH67, Erd75] conjectured for each p there exists a graph G , containing no K_4 , which has the Ramsey property “if the edges of G are p -colored then there exists a monochromatic triangles”. This conjecture was proved by Folkman [Fol70] for $p = 2$, and by Nešetřil and Rödl [NR76] for general $p \geq 3$. Let $f(2, 3, 4)$ denote the smallest integer n so that such that there is a K_4 -free graph G on n vertices with the above Ramsey property for $p = 2$. Erdős set a prize of \$100 for the challenge $f(2, 3, 4) \leq 10^{10}$. This reward was claimed by Spencer [Spe88, Spe89] in 1989, who proved that $f(2, 3, 4) < 3 \times 10^9$. Erdős then offered another \$100 prize (see [CL06a] page 46) for the new challenge $f(2, 3, 4) < 10^6$. Instead of probabilistic methods, which was used by Spencer and others, Dr. Lu [Lu07] took a novel approach — spectral analysis on a class of highly symmetric graphs. He received \$100 prize by proving $f(2, 3, 4) \leq 9697$. This approach has already triggered a new improvement $f(2, 3, 4) \leq 941$ by Dudek and Rödl [DR08].

Paper 47: Andrew King, Linyuan Lu, and Xing Peng, An fractional analogue of Brooks’ theorem, submitted.

Let $\Delta(G)$ be the maximum degree of a graph G . Brooks' theorem states that the only connected graphs with chromatic number $\chi(G) = \Delta(G) + 1$ are complete graphs and odd cycles. King, Lu, and Peng [KLP] proved a fractional analogue of Brooks' theorem. They classify all connected graphs G such that the fractional chromatic number $\chi_f(G)$ is at least $\Delta(G)$. These graphs are complete graphs, odd cycles, C_8^2 , $C_5 \boxtimes K_2$, and graphs whose clique number $\omega(G)$ equals the maximum degree $\Delta(G)$. Among the two sporadic graphs, the graph C_8^2 is the square graph of cycle C_8 while the other graph $C_5 \boxtimes K_2$ is the strong product of C_5 and K_2 . In fact, they proved a stronger result; if a connected graph G with $\Delta(G) \geq 4$ is not one of the graphs listed above, then we have $\chi_f(G) \leq \Delta(G) - \frac{2}{67}$. For $\Delta = 3$, $\Delta(G) = 3$, Heckman and Thomas [HT01] conjectured that $\chi_f(G) \leq 14/5$ if G is triangle-free. Hatami and Zhu [HZ09] proved $\chi_f(G) \leq 3 - \frac{3}{64}$ for any triangle-free graph G with $\Delta(G) \leq 3$. Lu and peng [LP] improved it to $\chi_f(G) \leq 3 - \frac{3}{43}$.

Paper 49: Linyuan Lu and Xing Peng, Monochromatic 4-term arithmetic progressions in 2-colorings of $\mathbb{Z}_n$, submitted.

In this paper, Lu and Peng studied the least monochromatic 4-term arithmetic progressions in 2-colorings of $\mathbb{Z}_n$. This paper is motivated by a recent result of Wolf [Wol10] on the minimum number of monochromatic 4-term arithmetic progressions (4-APs, for short) in $\mathbb{Z}_p$, where p is a prime number. Wolf [Wol10] proved that there is a 2-coloring of $\mathbb{Z}_p$ with 0.000386% fewer monochromatic 4-APs than random 2-colorings; the proof is probabilistic and using quadratic functional analysis. In the paper, Lu and Peng presented an explicit and simple construction of a 2-coloring with 9.3% fewer monochromatic 4-APs than random 2-colorings. This problem leads them to consider the minimum number of monochromatic 4-APs in $\mathbb{Z}_n$ for general n . They obtain both lower bound and upper bound on the minimum number of monochromatic 4-APs in $\mathbb{Z}_n$. Wolf [Wol10] proved that any 2-coloring of $\mathbb{Z}_p$ has at least $(1/16 + o(1))p^2$ monochromatic 4-APs. Lu and Peng improve this lower bound into $(7/96 + o(1))p^2$. The method for $\mathbb{Z}_n$ naturally apply to the similar problem on $[n]$. In 2008, Parillo, Robertson, and Saracino [PRS08] constructed a 2-coloring of $[n]$ with 14.6% fewer monochromatic 3-APs than random 2-colorings. In 2010, Butler, Costello, and Graham [BCL10] used a new method to construct a 2-coloring of $[n]$ with 17.35% fewer monochromatic 4-APs (and 26.8% fewer monochromatic 5-APs) than random 2-colorings. Lu-Peng's construction gives a 2-coloring of $[n]$ with 33.33% fewer monochromatic 4-APs (and 57.89% fewer monochromatic 5-APs) than random 2-colorings.

Citations, conferences, and services:

The impact of Dr. Lu's research is shown by 386 citations from 300 authors in AMS Math Reviews. Because of his interdisciplinary research component, some of his papers are often cited by computer scientists, physicists, sociologists, and even biologists. Those citations have not been counted by AMS Math Reviews. For example, the paper "Duplication Models for Biological Networks" [CLDG03], which is published in Journal of Computational Biology and has 190 citations, is not found at AMS Math Reviews. According to *Google Scholar*, Lu

has over 3000 citations in total. Nine of his papers exceed 100 citations each. The paper “A random graph model for massive graphs” [ACL00] has been cited 672 times. These shows a great impact on mathematics, computer science, physics, sociology, and biology.

Dr. Lu has frequently been invited to speak at many conferences. He gave 50-minutes invited talks at Atlanta Lecture Series in Combinatorics and Graph Theory IV (2011), at the 22nd Clemson mini-conference (2007), and twenty-two 25-minutes talks at AMS and SIAM conferences. Dr. Lu was invited to give colloquium/seminar talks at other Universities such as Harvard University, Georgia Institute of Technology, Tsinghua University, Georgia State University, and Nankai University. In particular, Dr. Lu was invited to give a series of six 90-minute talks on complex graphs in a workshop in China in 2008. These talks are based on the book “Complex Graphs and Networks” written by Lu and Chung [CL06a]. The organizers of this workshop try to promote this new research area in China.

Lu served an organizing committee member for “the 5th Workshop on Algorithms and Models for the Web-Graph” (WAW2007), La Jolla, December 11-12, 2007. He is the Program Committee member for WAW 2010-2012. He co-organized several sessions at AMS and SIAM sectional meetings. Dr. Lu is the *managing editor of Journal of Combinatorics* since 2009.

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