

$$P0 := - \frac{2 \cdot r^4 + 9 r^3 + 16 \cdot r^2 + 6 \cdot r + 2}{24 \cdot r \cdot (r + 1)^3} - \frac{1}{24} \frac{2 r^4 + 9 r^3 + 16 r^2 + 6 r + 2}{r (r + 1)^3} \quad (1)$$

$$P1 := - \frac{r^2 + 3 \cdot r + 1}{2 \cdot r \cdot (r + 1)^2} - \frac{1}{2} \frac{r^2 + 3 r + 1}{r (r + 1)^2} \quad (2)$$

$$P2 := - \frac{1}{2 \cdot r \cdot (r + 1)} - \frac{1}{2 r (r + 1)} \quad (3)$$

$$Q0 := \frac{6 + 24 \cdot r + 100 \cdot r^2 - 636 \cdot r^3 - 588 \cdot r^4 - 384 \cdot r^5 - 143 \cdot r^6 - 12 \cdot r^7 + 4 \cdot r^8}{1152 \cdot r^2 \cdot (r + 1)^6} - \frac{1}{1152} \frac{6 + 24 r + 100 r^2 - 636 r^3 - 588 r^4 - 384 r^5 - 143 r^6 - 12 r^7 + 4 r^8}{r^2 (r + 1)^6} \quad (4)$$

$$Q1 := \frac{6 + 32 \cdot r + 56 \cdot r^2 + 135 \cdot r^3 + 101 \cdot r^4 + 37 \cdot r^5 + 6 \cdot r^6}{48 \cdot r^2 \cdot (r + 1)^5} - \frac{1}{48} \frac{6 + 32 r + 56 r^2 + 135 r^3 + 101 r^4 + 37 r^5 + 6 r^6}{r^2 (r + 1)^5} \quad (5)$$

$$Q2 := \frac{20 + 90 \cdot r + 190 \cdot r^2 + 105 \cdot r^3 + 20 \cdot r^4}{48 \cdot r^2 \cdot (r + 1)^4} - \frac{1}{48} \frac{20 + 90 r + 190 r^2 + 105 r^3 + 20 r^4}{r^2 (r + 1)^4} \quad (6)$$

$$Q3 := \frac{5 + 15 \cdot r + 5 \cdot r^2}{12 \cdot r^2 \cdot (r + 1)^3} - \frac{1}{12} \frac{5 + 15 r + 5 r^2}{r^2 (r + 1)^3} \quad (7)$$

$$Q4 := \frac{1}{8 \cdot r^2 \cdot (r + 1)^2} - \frac{1}{8 r^2 (r + 1)^2} \quad (8)$$

$$\begin{aligned}
B := (n, h) &\rightarrow \frac{(n+h)!}{r^h} \cdot \left(1 + (P0 + h \cdot P1 + h^2 \cdot P2) \cdot \frac{r}{n} + (Q0 + h \cdot Q1 + h^2 \cdot Q2 + h^3 \cdot Q3 \right. \\
&\quad \left. + h^4 \cdot Q4) \cdot \frac{r^2}{n^2} + r^3 \cdot O\left(\frac{1}{n^3}\right) \right); \\
(n, h) &\rightarrow \frac{1}{r^h} \left((n+h)! \left(1 + \frac{(P0 + h P1 + h^2 P2) r}{n} \right. \right. \\
&\quad \left. \left. + \frac{(Q0 + h Q1 + h^2 Q2 + h^3 Q3 + h^4 Q4) r^2}{n^2} + r^3 O\left(\frac{1}{n^3}\right) \right) \right) \tag{9}
\end{aligned}$$

$$\begin{aligned}
Bstar := (n, h) &\rightarrow B(n, h-1) - B(n, h-2) + B(n, h-3) - B(n, h-4) + B(n, h-5) \\
&\quad - B(n, h-6) + B(n, h-7) + C \cdot B(n, h-8); \\
(n, h) &\rightarrow B(n, h-1) - B(n, h-2) + B(n, h-3) - B(n, h-4) + B(n, h-5) - B(n, h-6) \\
&\quad + B(n, h-7) + C B(n, h-8) \tag{10}
\end{aligned}$$

$$\begin{aligned}
sort \left(simplify \left(asympt \left(\frac{Bstar(n, 2)}{Bstar(n, 0)} + 2 \cdot n \cdot \frac{Bstar(n, 1) \cdot Bstar(n, -1)}{Bstar(n, 0)^2} + n \cdot (n-1) \cdot \frac{Bstar(n, -2)}{Bstar(n, 0)} \right. \right. \right. \\
\left. \left. - \frac{Bstar(n, 1)^2}{Bstar(n, 0)^2} - n^2 \cdot \frac{Bstar(n, -1)^2}{Bstar(n, 0)^2} - n \cdot \frac{Bstar(n, -1)}{Bstar(n, 0)} - (2 \cdot n + 1), \quad n, 5 \right) \right), \quad order \\
= (plex(n, r)) \Big);
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2} \frac{1}{(r+1)^4 r} \left(2 n r^3 + 6 n r^2 + 6 n r + 2 n - 2 r^6 + 2 O\left(\frac{1}{n}\right) r^5 - 6 r^5 + 8 O\left(\frac{1}{n}\right) r^4 \right. \\
\left. - 8 r^4 + 12 O\left(\frac{1}{n}\right) r^3 - 9 r^3 + 8 O\left(\frac{1}{n}\right) r^2 - 9 r^2 + 2 O\left(\frac{1}{n}\right) r - 2 r \right) \tag{11}
\end{aligned}$$

$$\begin{aligned}
sort \left(simplify \left(asympt \left(\frac{Bstar(n, 1)}{Bstar(n, 0)} - n \cdot \frac{Bstar(n, -1)}{Bstar(n, 0)}, \quad n, 2 \right) \right), \quad order = (plex(n, \\
r)) \Big);
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2} \frac{1}{(r+1)^2 r} \left(2 n r^2 + 4 n r + 2 n - 2 r^4 - 4 r^3 + 2 O\left(\frac{1}{n}\right) r^3 - 3 r^2 + 4 O\left(\frac{1}{n}\right) r^2 \right. \\
\left. - 2 r + 2 O\left(\frac{1}{n}\right) r \right) \tag{12}
\end{aligned}$$