

## 5.1 Completed Notes

### 5.1: Integers and the Operations of Addition and Subtraction

Notation: To avoid confusion between subtraction and negative numbers, a negative number will be denoted with a superscript -, such as  $-5$ . We read  $-x$  as "the opposite of  $x$ ".

Definition: The negative integers are the opposite of the positive integers, defined to be the value with the same distance from 0 but on its left side.

Example: Find the opposite of the following:

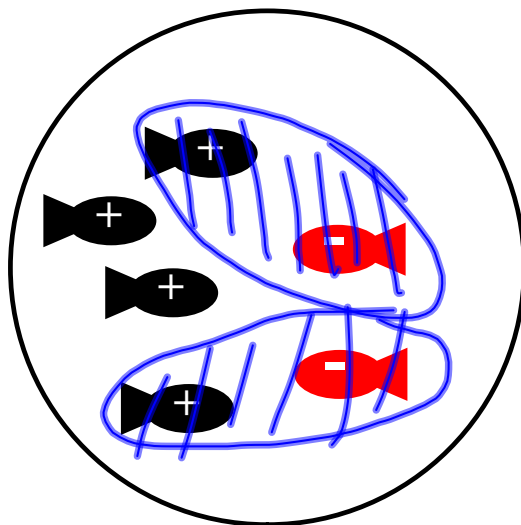
$$x = 2 \quad -2$$

$$x = -3 \quad 3$$

$$x = 0 \quad 0$$

Addition of Integers:

Chip Model: The chip model represents integers by using black chips for positive numbers and red chips for negative numbers. To make this more interesting, I use the concept of "posi-fish" and "nega-fish". When a posi-fish and a nega-fish are together, they slam into each other and destroy each other.

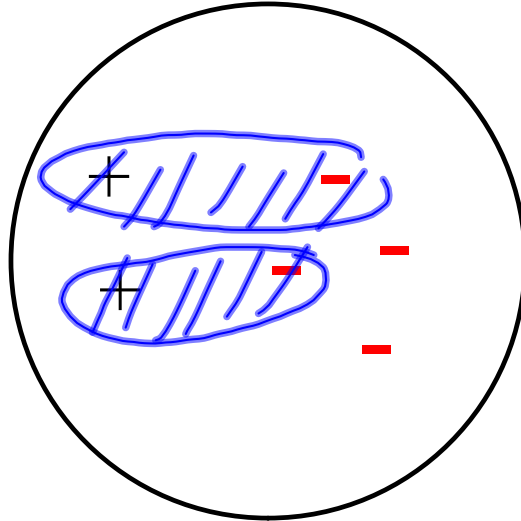


$$4 + -2 = 2$$

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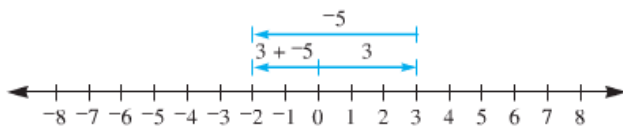
Charged Field Model: This model is virtually identical to the chip model, but we use just signs instead of images. The positive charges would "neutralize" the negative charges.

We will often use this model, but the other model is great for children.

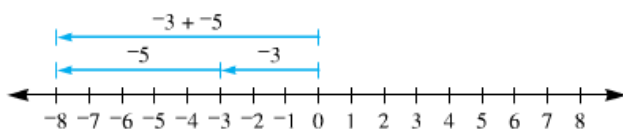


$$2 + -4 = -2$$

Number Line Model: You are standing on a number line facing the positive direction, then you move forward if adding a positive number or backward if adding a negative number.



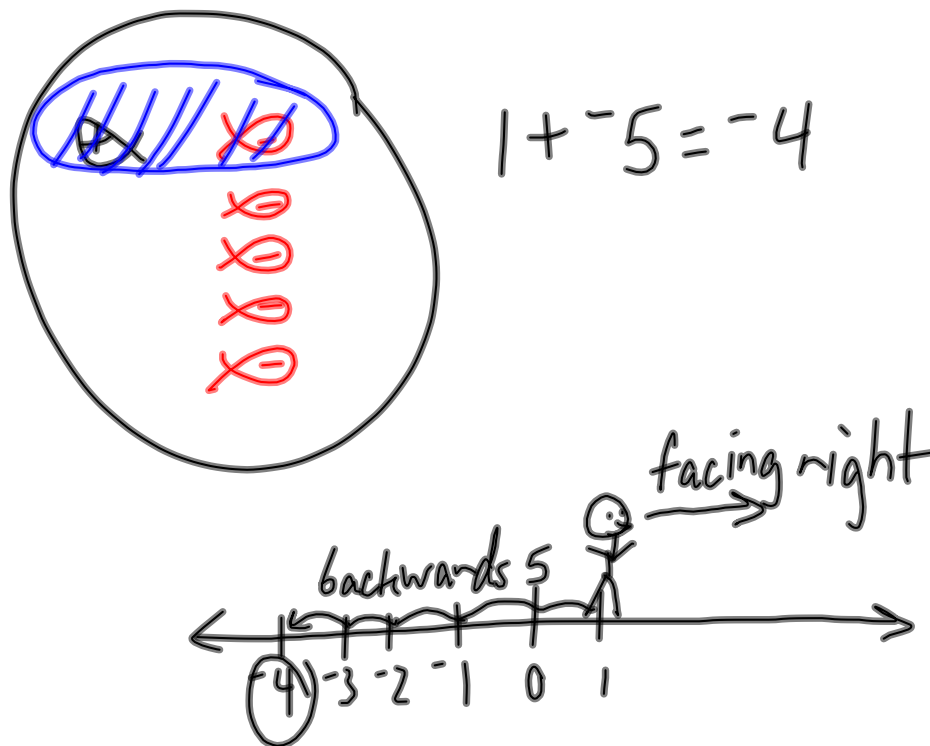
$$3 + -5 = -2$$



$$-3 + -5 = -8$$

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Example: Represent the problem  $1 + -5 = -4$  using each method.



Definition: The absolute value of  $x$ , denoted  $|x|$ , is the distance from 0 to  $x$ . Since distance is always positive, the absolute value of a number is just the positive version of that number. However, to define this more carefully:

$$|x| = x \text{ if } x \geq 0 \quad \text{and} \quad |x| = -x \text{ if } x < 0$$

Special Rules for Addition of Integers: The following set of rules is not in the text, but it can be taught to students who are struggling to speed up their calculations. This is NOT, however, a substitute for these models, as students should have a deep understanding of whether the solution is positive or negative before learning these.

1. If you are adding two numbers with the same sign, add their absolute values and keep the common sign.
2. If you are adding two numbers with different signs, subtract the number with the smaller absolute value from the number with the larger absolute value, then use the sign of the number with the larger absolute value.

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Properties of Integer Addition:

Closure, Commutative, Associative, and Identity still hold.

5. Additive Inverse Property: For every integer  $a$ , there is a unique integer  $-a$  such that  $a + -a = -a + a = 0$ .

Other Properties:

6.  $-(-a) = a$

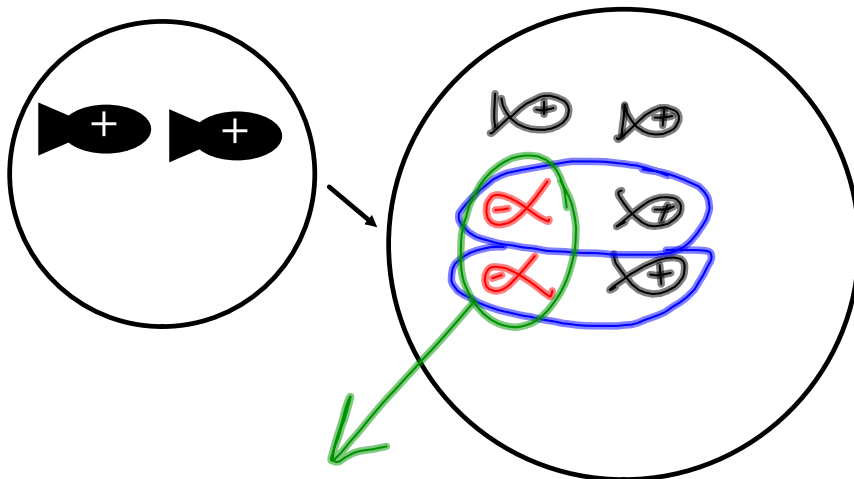
7.  $-(a + b) = -a + -b$

Subtraction of Integers:

Definition: For integers  $a$  and  $b$ ,  $a - b$  is the unique integer  $n$  such that  $a = b + n$ .

Chip Model: Using the same setup as the chip model for addition, we want to take away the described numbers, and this may require us to create paired posi-fish and nega-fish.

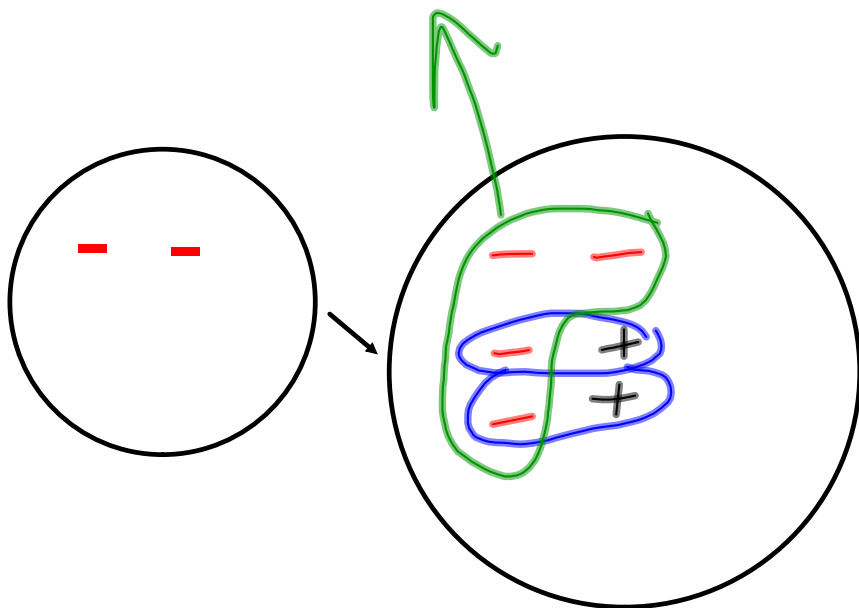
Example:  $2 - -2 = 4$



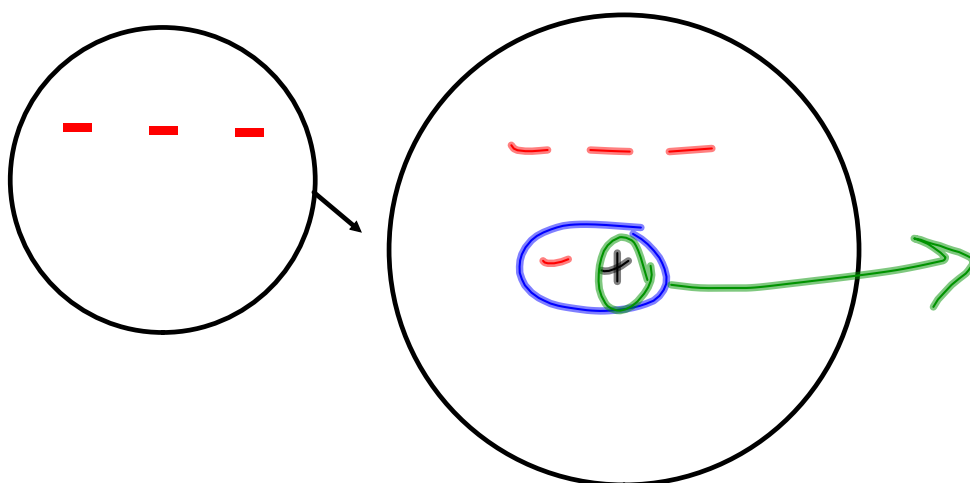
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Charged Field Model: We again just use signs instead of images.

Example:  $-2 - -4 = 2$

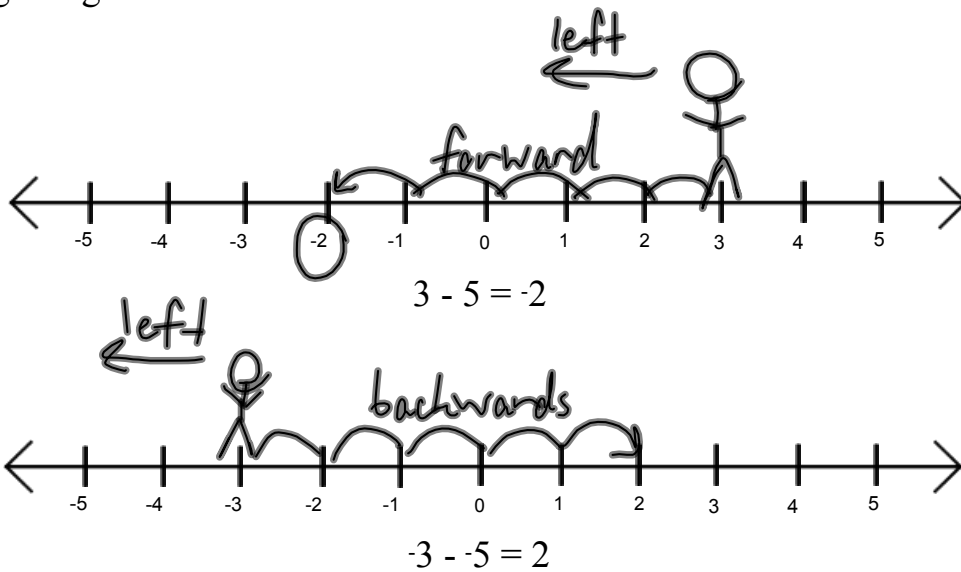


Example: Draw the chip/charged field model for  $3 - 1 = 4$ .



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Number Line Model: You are standing on a number line facing the **negative** direction, then you move forward if adding a positive number or backward if adding a negative number.



Remark: Again, after thorough understanding is given, this following equation is useful to help students calculate subtraction quickly.

$$a - b = a + -b$$

Use both models for  $3 - -2 = 5$

