Homework 9

Due Friday April 18, 2008 at the beginning of class.

13. Let $E = \mathbb{C}^3$, $V = \bigwedge^2 E$, and $\rho\colon \mathrm{GL}(E) \to \mathrm{GL}(V)$ be the holomorphic group homomorphism which exhibits V as a representation of $\mathrm{GL}(E)$. Exhibit the map $d\rho\colon \mathfrak{gl}(E) \to \mathfrak{gl}(V)$.

Note.

a. $\mathfrak{gl}(E)$ and $\mathfrak{gl}(V)$ both are equal to the set of 3×3 matrices with complex entries.

b. Your job job is to take an arbitrary matrix in $\mathfrak{gl}(E)$ and show me where it is sent in $\mathfrak{gl}(V)$. (Of course, you'll have to tell me what bases you are using.)

c. Your answer is actually a Lie Algebra homomorphism. Feel free to experiment with that phenomenon if you like – or not, as you wish.

d. Feel free to treat an E of larger dimension and/or a $V = E^\lambda$ for a more complicated partition than $\lambda = (1, 1)$.