Homework 3

Due Friday February 1, 2008 at the beginning of class.

5. Let the Symmetric group S_3 act on the vector space $\mathbb{C}^3 = \mathbb{C}x_1 \oplus \mathbb{C}x_2 \oplus \mathbb{C}x_3$ by $\sigma(x_i) = x_{\sigma(i)}$ for all $\sigma \in S_3$ and $1 \le i \le 3$. We noticed that $W_1 = \mathbb{C}(x_1+x_2+x_3)$ is an S_3-submodule of $\mathbb{C}^3$. Identify a complementary S_3-submodule W_2 of $\mathbb{C}^3$ with $W_1 \oplus W_2 = \mathbb{C}^3$ (as an internal direct sum). Is W_2 irreducible? What are the characters associated to each of the representations W_1, W_2, and $\mathbb{C}^3$? Please write down complete explanations.

Definition. Let V be a finite dimensional vector space over the field K. If V is a representation of the group G, then the *character* associated to V is the function $\chi\colon G \to K$ which is given by $g \mapsto \mathrm{tr}(g\colon V \to V)$. In this discussion tr stands for trace. The trace of a square matrix is the sum of the elements on its main diagonal. The function "multiplication by g" corresponds to a linear transformation from V to V; hence, a square matrix.