PRINT Your Name:____________________________________

Quiz for June 6, 2007

Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

(a) the positive real numbers,
(b) the integers,
(c) the nonzero real numbers.

Give a short justification for each answer.

ANSWER:

(a) FALSE. If x is a positive real number, then $y = \frac{\sqrt{x}}{2}$ is also a positive real number; but $y^2 = \frac{x}{4} < x$.

(b) TRUE. Take $x = -1$. (Notice that -1 is an integer.) Then x is less than y^2 for all integers y .

(c) TRUE. Take $x = -1$. (Notice that -1 is a nonzero real number.) Then x is less than y^2 for all (nonzero) real numbers y .