PRINT Your Name:__________________________________

Quiz for June 14, 2007

Consider a group of ten people in which each pair of individuals consists of two friends or two enemies. Prove that there are either three mutual friends or four mutual enemies in the group of ten people.

ANSWER: Let A be one of the people. The Pigeon Hole Principle ensures that A has at least 4 friends or at least 6 enemies. (Indeed, if A has less than 4 friends, all of the non-friends, and there at least six of them, are enemies of A.) We will examine each case carefully.

Case 1. A has at least 4 friends $\{B, C, D, E\}$ AND at least one pair from $\{B, C, D, E\}$ are friends. We are finished in this case because A together with the pair of friends from $\{B, C, D, E\}$ form a triangle of mutual friends.

Case 2. A has at least 4 friends $\{B, C, D, E\}$ AND every pair from $\{B, C, D, E\}$ is a pair of enemies. We are finished in this case because $\{B, C, D, E\}$ is a four-some of mutual enemies.

Case 3. A has at least 6 enemies $\{F, G, H, I, J, K\}$. The result from class tells us that $\{F, G, H, I, J, K\}$ contains a triangle of mutual friends or a triangle of mutual enemies. Case 3 is the case where $\{F, G, H, I, J, K\}$ contains a triangle of mutual friends. Once again we are finished because we have exhibited a triangle of mutual friends.

Case 4. A has at least 6 enemies $\{F, G, H, I, J, K\}$, and the enemy set $\{F, G, H, I, J, K\}$ contains a triangle of mutual enemies. We are also finished in this case because A together with the three mutual enemies from $\{F, G, H, I, J, K\}$ forms a four-some of of mutual enemies.