PRINT Your Name:____

Quiz for March 23, 2006

How many integer solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$

if each $x_i \geq 2$?

ANSWER: Let $y_i = x_i - 2$. The given problem is equivalent to the problem of solving

$$y_1 + y_2 + y_3 + y_4 + y_5 = 11$$

with each y_i a non-negative integer. I think of this problem as I have five bins labeled y_1, y_2, $\ldots$, y_5 arranged in a straight line, and an arm full of 11 ones. Each solution to the equation consists of a work order of 11 drops and 4 switches. There are $\boxed{\binom{15}{4}}$ such work orders. I walk to the first bin. If the first instruction is drop, I put a one in the bin, then I read the second instruction. Whenever the instruction says swith, I move to the next bin. I continue until I have dropped all of my ones.