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Quiz for January 19, 2006

Goldbach's conjecture states that every even integer greater than 2 is the sum of two primes. Prove that Goldbach's conjecture is equivalent to the statement that every integer greater than 5 is the sum of three primes.

ANSWER:

Assume the original conjecture. Prove the alternate form. Let n be an integer greater than 5. If n is even, then $n-2$ is an even integer greater than 2 and Goldbach's conjecture ensures that there exist prime numbers p and q with $p+q=n-2$. Thus, $p+q+2=n$ and the conclusion of the alternate form holds for n. If n is odd, then $n-3$ is an even integer greater than 2. Once again Goldbach's conjecture ensures that there exist prime numbers p and q with $p+q=n-3$. Thus, $p+q+3=n$. In any event, n is the sum of three primes.

Assume the alternate form. Prove the original conjecture. Let $n>2$ be an even integer. We see that $n+2$ is an arbitrary integer greater than 5. The alternate form of the conjecture ensures that there exist prime numbers p, q, and r with $n+2=p+q+r$. We notice that at least one of the numbers p, q, and r must be even (because three odd numbers add up to an odd number and $n+2$ is even). The only even prime number is 2. So one of the three prime numbers p, q or r is equal to 2. Re-label, if necessary, in order to have $r=2$. We now subtract 2 from each side of $n+2=p+q+2$ to see that $n=p+q$.