Math 574, Exam 4, Spring 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 7 problems. Write in complete sentences **No Calculators.**

YOU MUST JUSTIFY YOUR ANSWERS.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website a few hours after the exam is finished.

1. (7 points) Express the sum $\displaystyle\sum_{k=0}^{r}\binom{n+k}{k}$ as a single binomial coefficient. PROVE your answer. (If you quote a formula we did in class, then the prove the formula.)

2. (7 points) How many paths are there from $(0,0)$ to $(8,5)$ on the xy-plane if each path consists of a series of steps, where each step is either a move one unit to the right or a move one unit up. (No moves to the left or downward are allowed.)

3. (7 points) What is the coefficient of $x^4y^2zw^3$ in the expansion of $(x+y+z+w)^{10}$?

4. (7 points) How many different terms are there in the expansion of $(x+y+z+w)^{10}$ (after collecting the common terms)?

5. (7 points) Consider this VARIATION of the Tower of Hanoi problem. There are three towers in a ROW: tower A, tower B, and tower C. There are n disks of different sizes stacked on tower A. One must move all n disks to tower C. One may NEVER place a bigger disk on top of a smaller disk. In the present problem, one may move a disk only to an ADJACENT tower. Let a_n be the minimum number of moves needed to transfer a stack of n disks from tower A to tower C. Find a_1 , a_2 , a_3 . Find a recurrence relation for $a_1, a_2, a_3, \dots$.

6. (7 points) Find a recurrence relation for the number of strings made from 0's, 1's, and 2's that contain two consecutive zeros.

7. (8 points) Solve the recurrence relation $a_n = -a_{n-1} + 6a_{n-2} - 4n + 23$ with $a_0 = 0$ and $a_1 = -6$. **CHECK your answer.**