Math 574, 1992, Exam 3

PRINT Your Name:____________________

There are 6 problems. The exam is worth a total of 100 points.

SHOW your work. $\boxed{CIRCLE}$ your answer.

1. (16 points) How many integer solutions are there to the inequality

$$x_1 + x_2 + x_3 + x_4 \le 14,$$

with $0 \le x_i$, for all i?

2. (16 points) How many $40-$ digit quaternary sequences have exactly 12 zeros? (A *quaternary* sequence is a sequence made out of 0's, 1's, 2's, and 3's.)

3. (17 points) Write

$$\binom{100}{0} + \binom{101}{1} + \binom{102}{2} + \binom{103}{3} + \cdots + \binom{997}{897} + \binom{998}{898} + \binom{999}{899} + \binom{1000}{900}$$

as one binomial coefficient. Explain your work.

4. (17 points) Write

$$\binom{500}{0}\binom{1000}{300} + \binom{500}{1}\binom{1000}{299} + \binom{500}{2}\binom{1000}{298} + \binom{500}{3}\binom{1000}{297} + \cdots$$
$$\cdots + \binom{500}{297}\binom{1000}{3} + \binom{500}{298}\binom{1000}{2} + \binom{500}{299}\binom{1000}{1} + \binom{500}{300}\binom{1000}{0}$$

as one binomial coefficient. Explain your work.

5. (17 points) Find a recurrence relation for the number of ways to arrange identical Toyotas, identical Hondas, identical Fords, and identical Cadillacs into n spaces if Toyotas and Hondas each take up one space, Fords take up two spaces, and Cadillacs take up three spaces. Do not forget to give sufficient initial conditions.

6. (17 points) What is the probability that a random arrangement of the letters in MISSISSIPPI has no consecutive vowels?