PRINT Your Name:____________________________________

Quiz for June 8, 2004

Prove that if the sequence $\{a_n\}$ converges to a, then the sequence $\{|a_n|\}$ converges to $|a|$. Is the converse true? Prove or give a counter example.

ANSWER: Given $\varepsilon > 0$, there exists n_0 such that whenever $n_0 \le n$, then $|a_n - a| \le \varepsilon$. Notice that $||a_n| - |a|| \le |a_n - a|$; therefore, the very same n_0 has the property that whenever $n_0 \le n$, then $||a_n| - |a|| \le \varepsilon$. We have established that the sequence $\{|a_n|\}$ converges to $|a|$.

The converse is false. For example the sequence if $a_n = (-1)^n$, then the sequence $\{|a_n|\}$ converges to 1; but the sequence $\{a_n\}$ diverges.