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Quiz for June 5, 2006

Give a complete proof that the limit of the sequence $\{\frac{n^2+1}{2n^2}\}$ is $\frac{1}{2}$. In other words, given $\varepsilon > 0$, you are required to find a formula for n_0 with the property that:

$$(\,*\,) \qquad \textit{whenever } n > n_0\textit{, then:} \qquad \left|\frac{n^2+1}{2n^2} - \frac{1}{2}\right| < \varepsilon.$$

Your formula for n_0 will probably involve ε. I expect a complete, coherent argument that $(\,*\,)$ holds. Write in complete sentences.

ANSWER: Let n_0 be any fixed integer with $\sqrt{\frac{1}{2\varepsilon}} < n_0$. (Of course, I did some scratch work to know this good choice for n_0.) Suppose n is any integer with $n_0 < n$. Both sides of the inequality

$$\sqrt{\frac{1}{2\varepsilon}} < n$$

are positive; so we may square both sides to obtain the inequality:

$$\frac{1}{2\varepsilon} < n^2.$$

Multiply by the positive number $\frac{\varepsilon}{n^2}$ to see that $\frac{1}{2n^2} < \varepsilon$. We are finished because

$$\left|\frac{n^2+1}{2n^2} - \frac{1}{2}\right| = \left|\frac{n^2}{2n^2} + \frac{1}{2n^2} - \frac{1}{2}\right| = \frac{1}{2n^2} < \varepsilon.$$