PRINT Your Name:____________________________________

Quiz for June 22, 2004

Find a countable collection $\{K_n\}_{n=1}^{\infty}$ of compact subsets of $\mathbb{R}$ such that the union $\bigcup_{n=1}^{\infty} K_n$ is not compact.

ANSWER: For each natural number n, let K_n be the closed interval $[-n, n]$. The Heine-Borel Theorem tells us that K_n is compact. The union $\bigcup_{n=1}^{\infty} K_n$ is equal to all of $\mathbb{R}$, which is unbounded; and therefore not compact. (We saw in class that unbounded sets are never compact.)