PRINT Your Name:__________________________________

Quiz for June 21, 2005

Let $K=\{0\}\cup\{\frac{1}{n}\mid n=1,2,3,\dots\}$. Prove directly (using the definition) that K is compact.

Let $\mathcal{U}=\{U_a\mid a\in A\}$ be an open cover of K. The number 0 is in K; so, 0 is in U_{a_0}, for some $a_0\in A$. The set U_{a_0} is open; so there exists a positive ε with $N_\varepsilon(0)\subseteq U_{a_0}$. If n_0 is a large enough integer, then $\frac{1}{n_0}<\varepsilon$. It follows that for all integers n, with $n\ge n_0$, $\frac{1}{n}\in U_{a_0}$. Only finitely many elements of K are outside U_{a_0}; these elements are some subset of $\{\frac{1}{1},\frac{1}{2},\frac{1}{3},\dots,\frac{1}{n_0-1}\}$. If $1\le i\le n_0-1$, then there exists $a_i\in A$ with $\frac{1}{i}\in U_{a_i}$. Thus, the finite subset $\{U_{a_0},U_{a_1},U_{a_2},\dots,U_{a_{n_0-1}}\}$ of $\mathcal{U}$, covers K.