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**Quiz for June 1, 2006**

Let  $X$ ,  $Y$ , and  $Z$  be sets. Suppose that  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are one-to-one functions. Prove that the function  $g \circ f: X \rightarrow Z$  is one-to-one.

**ANSWER:** Let  $x_1$  and  $x_2$  be elements of  $X$  with  $(g \circ f)(x_1) = (g \circ f)(x_2)$ . We must prove that  $x_1 = x_2$ .

We are given  $g(f(x_1)) = g(f(x_2))$ . The function  $g$  is one-to-one; hence, the elements  $f(x_1)$  and  $f(x_2)$  in  $Y$  are equal. The function  $f$  is one-to-one; hence, the elements  $x_1$  and  $x_2$  in  $X$  are equal.