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Quiz for May 31, 2005

Let E be a nonempty subset of $\mathbb{R}$ that is bounded above, and set

$$U = \{\beta \in \mathbb{R} \mid \beta \text{ is an upper bound of } E\}.$$

Prove that $\sup E = \inf U$.

ANSWER: Let $\alpha = \sup E$ and $\gamma = \inf U$.

$\gamma \le \alpha$: The number α is an upper bound of E; and therefore, $\alpha \in U$. On the other hand, γ is a lower bound for U; so, $\gamma \le \alpha$.

We finish the argument by showing that $\gamma < \alpha$ is impossible. Suppose $\gamma < \alpha$. The number α is the supremum of E, and $\gamma < \alpha$; hence, there is an element $e \in E$ with $\gamma < e$. On the other hand, γ is the infimum of U, and $\gamma < e$; so, there is an element $u \in U$ with $u < e$. This is a contardiction because u is an upper bound of the set E and e is an element of E with $u < e$.

We have shown that $\gamma \le a$ and γ is not less than α . The only remaining possibility is $\gamma = \alpha$.