Math 554, Exam 1, Summer 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. **Leave room on the upper left hand corner of each page for the staple.** Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

There are 7 problems. Problem 1 is worth 8 points. Each of the other problems is worth 7 points. The exam is worth a total of 50 points.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website later this afternoon.

1. State the least upper bound axiom of the real numbers. **Use complete sentences.** Include everything that is necessary, but nothing more.

2. Define "limit of a sequence". **Use complete sentences.** Include everything that is necessary, but nothing more.

3. Suppose X and Y are sets, $f\colon X \to Y$ and $g\colon Y \to X$ are functions, and $g \circ f$ is the identity function on X. (In other words, $g(f(x)) = x$ for all $x \in X$.)
   (a) Does the function g have to be one-to-one? If yes, **prove** it. If no, give a **counter example**. Write in **complete sentences**.
   (b) Does the function g have to be onto? If yes, **prove** it. If no, give a **counter example**. Write in **complete sentences**.

4. Suppose that $\{a_n\}$ is a sequence which converges to a and $\{b_n\}$ is a sequence that converges to b. Prove that $\{a_n + b_n\}$ is a sequence which converges to $a + b$. **I expect a complete, coherent argument. Write in complete sentences.**

5. Prove that between any two real numbers there exists an irrational number. Give a **complete** proof. Write in **complete sentences**. If you quote some result we did in class, be sure to quote the complete result.

6. Consider the sequence $\{a_n\}$ with $a_1 = \sqrt{12}$, and $a_n = \sqrt{12 + a_{n-1}}$ for $n \geq 2$. Prove that the sequence $\{a_n\}$ converges. Find the limit of the sequence $\{a_n\}$. Write in **complete sentences**.

7. Let $\{a_k\}$ be a sequence of real numbers. For each natural number n, let

$$s_n = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

Suppose that the sequence $\{a_k\}$ converges to the real number a. Prove that the sequence $\{s_n\}$ also converges to a. Give a complete ε style proof. Write in **complete sentences**.