Math 554, Exam 2, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

There are 9 problems. Problems 1 through 4 are worth 5 points each. Problems 5 through 9 are worth 6 points each. The exam is worth a total of 50 points.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Define *upper bound*. **Use complete sentences.** Include everything that is necessary, but nothing more.

2. Define *supremum*. **Use complete sentences.** Include everything that is necessary, but nothing more.

3. Define *the sequence converges*. **Use complete sentences.** Include everything that is necessary, but nothing more.

4. State the Nested Interval Property. **Use complete sentences.** Include everything that is necessary, but nothing more.

5. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers. Suppose that $\{a_n\}$ converges to the real number a and $\{b_n\}$ converges to the real number b. Prove that the sequence $\{a_nb_n\}$ converges to ab. ("We did this in class" is not a satisfactory answer. I expect a complete, coherent proof.)

6. Give an example of a set X and a function $f\colon X \to X$ with f one-to-one, but f not onto.

7. Find $\bigcap\limits_{n=1}^{\infty} [-n, n]$.

8. Suppose that A and B are non-empty sets of real numbers. Suppose further that 1 is a lower bound for both A and B. Let

$$C = \{ab \mid a \in A \text{ and } b \in B\}.$$

Prove $\inf C = (\inf A)(\inf B)$.

9. Let S be a set of real numbers. Let p be a limit point of S. Prove that there exists a sequence a_n IN S which converges to p.