Math 554, Exam 1, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc; although, by using enough paper, you can do the problems in any order that suits you.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

There are 7 problems. Problems 1 through 5 are worth 5 points each. Problem 6 is worth 10 points. Problem 7 is worth 15 points. The exam is worth a total of 50 points.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Define *upper bound*. **Use complete sentences.** Include everything that is necessary, but nothing more.

2. Define *supremum*. **Use complete sentences.** Include everything that is necessary, but nothing more.

3. State the least upper bound axiom of the real numbers. **Use complete sentences.** Include everything that is necessary, but nothing more.

4. State the Archimedian property of the real numbers. **Use complete sentences.** Include everything that is necessary, but nothing more.

5. Let S be the following set of order pairs:
$$S = \{(a,b) \mid a,b \in \mathbb{N}\}.$$
Is S a countable set? If yes, exhibit a one-to-one and onto function $f\colon \mathbb{N} \to S$. If no, why not?

6. Suppose $f\colon X \to Y$ and $g\colon Y \to X$ are functions and that $g \circ f$ is the identity function on X. (In other words, $g(f(x)) = x$ for all $x \in X$.)
   (a) Does the function f have to be one-to-one? If yes, **prove** it. If no, give a **counter example**.
   (b) Does the function f have to be onto? If yes, **prove** it. If no, give a **counter example**.

7. Suppose that A and B are non-empty sets of real numbers with $12 \le a \le 20$ for all $a \in A$ and $2 \le b \le 4$ for all $b \in B$. Let
$$C = \{\tfrac{a}{b} \mid a \in A \text{ and } b \in B\}.$$
   (a) What is an upper bound for C? Prove your answer.
   (b) Give a formula for $\sup C$ in terms of $\sup A$, $\sup B$, $\inf A$, and $\inf B$.
   (c) Prove your answer to (b).