Math 554, Exam 4, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

There are 8 problems. Problems 1 and 2 are worth 7 points each. Problems 3 through 8 are worth 6 points each. The exam is worth a total of 50 points.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. For each natural number n, let C_n be a closed set in $\mathbb{R}$. Is the intersection $\bigcap_{n=1}^{\infty} C_n$ always a closed set? If yes, prove the result. If no, give a counterexample.

2. For each natural number n, let C_n be a closed set in $\mathbb{R}$. Is the union $\bigcup_{n=1}^{\infty} C_n$ always a closed set? If yes, prove the result. If no, give a counterexample.

3. Define *open set*. Use complete sentences.

4. Define *compact*. Use complete sentences.

5. State the Heine-Borel Theorem.

6. Prove the Heine-Borel Theorem.

7. Let $f(x) = \begin{cases} 2x-1 & \text{if } x \le 2 \\ 2x+1 & \text{if } 2 < x. \end{cases}$ What is $\lim_{x\to 2} f(x)$? Prove your answer.

8. Let $f(x) = \begin{cases} 2x-1 & \text{if } x \le 2 \\ 2x+1 & \text{if } 2 < x. \end{cases}$ What is $\lim_{x\to 3} f(x)$? Prove your answer.