Math 554, Exam 1, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.

There are 7 problems. Problems 1 through 6 are worth 7 points each. Problem 7 is worth 8 points. The exam is worth a total of 50 points.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Define *upper bound*.

2. Define *supremum*.

3. State the least upper bound axiom.

4. Let $f\colon X \to Y$ and $g\colon Y \to Z$ be functions with f onto and g onto, prove that the function $g \circ f\colon X \to Z$ is onto.

5. Let x and y be real numbers with $0 < x$. Prove that there exists a positive integer N with $Nx > y$. (I want you to give a complete proof of this result. I want more that its name. I want more than the statement that "we did this in class".)

6. Exhibit a one-to-one and onto function f from the open inteval $(3,4)$ to the open interval $(7,12)$.

7. Let A and B be non-empty sets of real numbers. Suppose that $\inf A = 2$, $\sup A = 6$, $\inf B = 4$ and $\sup B = 30$. Let
$$C = \{\tfrac{a}{b} \mid a \in A, \text{ and } b \in B\}.$$
What is $\inf C$? Give a complete proof that your answer is correct.