

(15) Given  $\epsilon > 0$ ,

$$|f(x)g(x) - AB| = |f(x)g(x) - Ag(x) + Ag(x) - AB| \leq |g(x)| |f(x) - A| + |A| |g(x) - B|$$

When I choose  $\delta$  I will insist that

$$0 < |x - p| < \delta \Rightarrow |g(x) - B| < 1$$

$$\text{Then } |g(x)| - |B| < |g(x) - B| < 1$$

$$\text{so } |g(x)| < |B| + 1$$

With this in mind

$$|f(x)g(x) - AB| \leq (|B| + 1) |f(x) - A| + (|A| + 1) |g(x) - B|$$

$$\text{Take } \delta_1 \text{ so that } 0 < |x - p| < \delta_1 \Rightarrow |f(x) - A| < \frac{\epsilon}{2(|B| + 1)}$$

$$\text{Take } \delta_2 \text{ so that } 0 < |x - p| < \delta_2 \Rightarrow |g(x) - B| < \frac{\epsilon}{2(|A| + 1)}$$

$$\text{I assume that } \frac{\epsilon}{2(|A| + 1)} < 1.$$

Let  $\delta = \min\{\delta_1, \delta_2\}$ . Then

$$0 < |x - p| < \delta \Rightarrow |f(x)g(x) - AB| \leq (|B| + 1) \frac{\epsilon}{2(|B| + 1)} + (|A| + 1) \frac{\epsilon}{2(|A| + 1)}$$

$$= \frac{\epsilon}{2} + \frac{\epsilon}{2} < \epsilon$$

$$\therefore \lim_{x \rightarrow p} f(x)g(x) = AB$$