

⑧ Let $x < y$ be real numbers. The Corollary to ④ the Archimedean Property of the real numbers tells us that there exists an integer n with

$$\frac{1}{n} < y - x$$

Case 1 If x is irrational, then $x + \frac{1}{n}$ is an irrational number with $x < x + \frac{1}{n} < y$

Case 2 If x is rational, then $x + \frac{\sqrt{2}}{2} \frac{1}{n}$ is an irrational number with $x < x + \frac{\sqrt{2}}{2} \frac{1}{n} < y$