Every number which we have written down is positive so we combine the inequalities to see (3)

$$ab < (\alpha+\varepsilon_1)(\beta+\varepsilon_2)$$

We finish the argument by making ~~a~~ ε_1 and ε_2 small enough to force $(\alpha+\varepsilon_1)(\beta+\varepsilon_2) < \gamma$

Well $(\alpha+\varepsilon_1)(\beta+\varepsilon_2) = \alpha\beta + \varepsilon_1\beta + \varepsilon_2\alpha + \varepsilon_1\varepsilon_2$

I know that $\alpha\beta < \gamma$ so I'll just make

$\varepsilon_1\beta < \frac{\gamma-\alpha\beta}{3}$ and $\alpha\varepsilon_2 < \frac{\gamma-\alpha\beta}{3}$ and $\varepsilon_1\varepsilon_2 < \frac{\gamma-\alpha\beta}{3}$. (✗)

Here is how I'll do that:

Notice that $\frac{\gamma-\alpha\beta}{3}\frac{1}{\beta}$, $\frac{\gamma-\alpha\beta}{3}\frac{1}{\alpha}$, 1, and $\frac{\gamma-\alpha\beta}{3}$

are all positive numbers so I'll just pick

$\varepsilon_1=\varepsilon_2$ positive but less than all of them. Now (✗)

holds so

$$ab < (\alpha+\varepsilon_1)(\beta+\varepsilon_2) = \alpha\beta+\varepsilon_1\beta+\varepsilon_2\alpha+\varepsilon_1\varepsilon_2 < \alpha\beta + 3\,\frac{\gamma-\alpha\beta}{3} = \gamma$$

Thus γ is not a lower bound for $A\cdot B$ for any γ with $\alpha\beta<\gamma$. We conclude that $\alpha\beta = \inf(A\cdot B)$