Math 544 Exam 2 Summer 2000

Use the paper provided. Put your name on the front of the first page and the back of the last page. Problems 7 and 8 are worth 7 points each. The other problems are worth 6 points each.

1. Define “limit of a sequence”.

2. Define “one-to-one”.

3. Define “countable”.

4. Prove that the open interval $(0,1)$ and the open interval $(-1,2)$ have the same cardinality.

5. Prove that the open interval $(0,1)$ is uncountable. (I do want to see a proof here. “We proved this in class” is not an acceptable answer.)

6. Let A and B be non-empty sets, and let $f\colon A \to B$ and $g\colon B \to A$ be functions. Suppose that $g \circ f$ is the identity function on A. (In other words, $g(f(a)) = a$ for all a in A.)
   (a) Prove that f is one-to-one.
   (b) Prove that g is onto.
   (c) Give an example to show that f does not have to be onto. (Your example can be very small.)

7. Let $\{a_n\}$ and $\{b_n\}$ be sequences. Suppose that $\{a_n\}$ converges to a and $\{b_n\}$ converges to b. Prove that the sequence $\{a_n + b_n\}$ converges to $a+b$. (I do want to see a proof here. “We proved this in class” is not an acceptable answer.)

8. Let $\{a_n\}$ be a sequence of real numbers with $-100 < a_n < 100$ for all n.
   (a) Suppose the sequence $\{a_n\}$ converges to a. Prove that the sequence $\{a_n^2\}$ converges to a^2. (I want to see a proof here. Either **prove** the statement I have written or **prove** a more general statement and deduce this statement from it. “This is a special case of xxx, which was proved in class”, is not an acceptable answer.)
   (b) Give an example of a sequence $\{a_n\}$ for which $\{a_n^2\}$ converges, but $\{a_n\}$ does not converge.