Math 544 Exam 1 Summer 2000

Use the paper provided. Put your name on the front of the first page and the back of the last page. Problems 7 and 8 are worth 7 points each. The other problems are worth 6 points each.

1. Define "lower bound".

2. Define "infimum".

3. State the Least Upper Bound Property of the real numbers.

4. State the Archimedian property of the real numbers.

5. Let E be a non-empty subset of the real numbers which is bounded below and let
$$L = \{\ell \in \mathbb{R} \mid \ell \text{ is a lower bound for } E\}.$$
Prove that $\sup L = \inf E$.

6. Give an example of a non-empty subset E of real numbers such that E is bounded above and bounded below and $\inf E \in E$, but $\sup E \notin E$.

7. Let A and B be non-empty subsets of real numbers. Suppose that every element of A and every element of B is at least 1. Prove
$$\inf(A \cdot B) = (\inf A)(\inf B).$$

8. Prove that between any two real numbers there exists an irrational number.