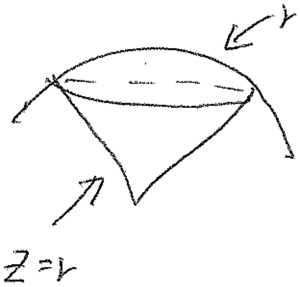
2000 Exam 2

1. Let $u = \frac{x}{a}$ $v = \frac{y}{b}$

$$\iint_{\substack{\text{inside} \\ (\frac{x}{a})^2+(\frac{y}{b})^2=1}} \left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2 \, dx\,dy = \iint_{u^2+v^2=1} u^2+v^2 \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} du\,dv = ab\int_0^{2\pi}\int_0^1 r^3\,dr\,d\theta$$

$$= 2\pi ab \left.\frac{r^4}{4}\right]_0^1 = \frac{\pi ab}{2}$$

2.



intersection is $2r^2=1$ $r=\frac{1}{\sqrt{2}}$

$$Vol = \int_0^{2\pi}\int_0^{\frac{1}{\sqrt{2}}}\int_r^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta$$

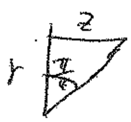
$$= 2\pi\int_0^{\frac{1}{\sqrt{2}}} r\sqrt{1-r^2} - r^2\,dr = 2\pi\left(-\frac{1}{3}(1-r^2)^{\frac{3}{2}} - \frac{r^3}{3}\right)\Bigg]_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{2\pi}{3}\left(-\left(\frac{1}{2}\right)^{\frac{3}{2}} - \left(\frac{1}{2}\right)^{\frac{3}{2}} + 1\right) = \frac{2\pi}{3}\left(-2\frac{1}{2\sqrt{2}} + 1\right) = \frac{2\pi}{3}\left(1 - \frac{1}{\sqrt{2}}\right)$$

or

$$Vol = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 \rho^2 \sin\varphi\,d\rho\,d\varphi\,d\theta = 2\pi\left(-\cos\varphi\right)\Big]_0^{\frac{\pi}{4}} \left.\frac{\rho^3}{3}\right]_0^1$$

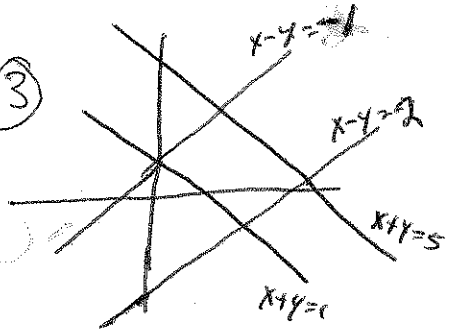
the same



$r = z$ is $\frac{\pi}{4} = \varphi$

$$= \frac{2\pi}{3}\left(1 - \frac{\sqrt{2}}{2}\right)$$

3.



Let $u = x-y$
$v = x+y$

$\frac{u+v}{2} = x$

$\frac{v-u}{2} = y$

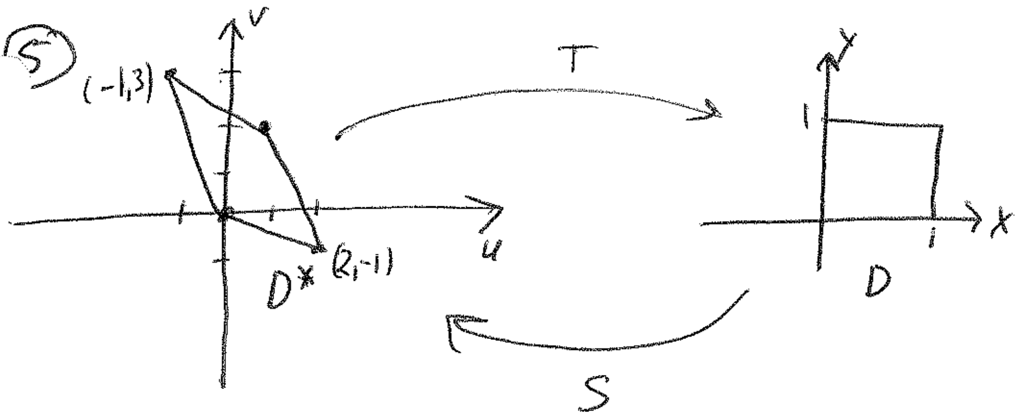
$$\text{integral} = \int_1^5\int_{-1}^{2} v^3 e^u \left|\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix}\right| du\,dv$$

$$= \frac{1}{2}\left.\frac{v^4}{4}\right]_1^5 \left. e^u \right]_{-1}^{2}$$

$$= \frac{1}{8}\left(5^4-1\right)\left(-\frac{1}{e}+e^2\right)$$

④ $\int_0^{2\pi}\int_0^1 re^{r^2}\,dr\,d\theta = 2\pi \, \frac{1}{2} e^{r^2}\Big]_0^1 = \pi(e-1)$

⑤



S is multiplication by $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$ T is mult. by $\frac{1}{5}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$T(u,v) = \left(\frac{1}{5}(3u+v), \frac{1}{5}(u+2v)\right)$

<u>check</u> $T(2,-1) = (1,0)$

$T(-1,3) = (0,1)$

$T(1,2) = (1,1)$