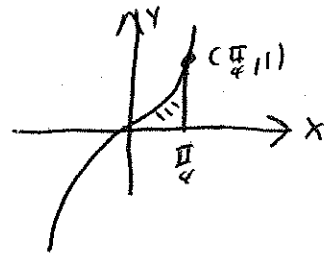
① $$\int_0^1 \int_{\tan^{-1}y}^{\frac{\pi}{4}} \sec^5 x \, dx\, dy = \int_0^{\frac{\pi}{4}} \int_0^{\tan x} \sec^5 x \, dy\, dx = \int_0^{\frac{\pi}{4}} \sec^5 x \tan x \, dx$$

$\tan^{-1} y = x$

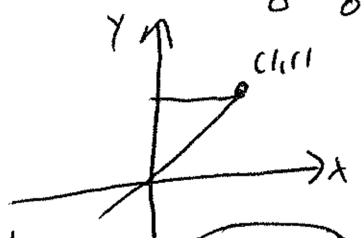
means $y = \tan x$



$$= \int_0^{\frac{\pi}{4}} \sec^4 x \sec x \tan x \, dx = \frac{\sec^5 x}{5}\Big]_0^{\frac{\pi}{4}} = \frac{1}{5}\left(\sec^5\left(\tfrac{\pi}{4}\right) - \sec^5(0)\right)$$

$$= \frac{1}{5}\left(\left(\sqrt{2}\right)^5 - 1\right)$$

② $$\int_0^1 \int_x^1 e^{-y^2} \, dy\, dx = \int_0^1 \int_0^y e^{-y^2} \, dx\, dy = \int_0^1 y e^{-y^2} \, dy$$

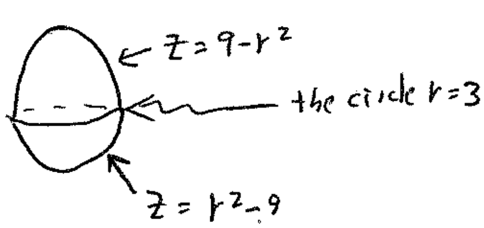


$$= -\frac{1}{2} e^{-y^2}\Big]_0^1 = -\frac{1}{2}\left(\frac{1}{e} - 1\right)$$

③ $$\int_1^2 \int_1^x e^x \, dy\, dx = \int_1^2 xe^x - e^x \, dx = xe^x - e^x - e^x\Big]_1^2$$

$$= 2e^2 - e^2 - e^2 - (e - e - e) = e$$

④

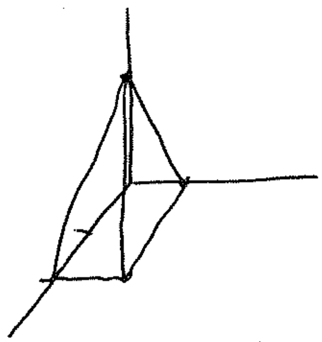


$$Vol = \int_0^{2\pi} \int_0^3 \int_{r^2-9}^{9-r^2} r \, dz\, dr\, d\theta = 2\int_0^{2\pi} \int_0^3 9r - r^3 \, dr\, d\theta$$

$$= 4\pi \left(9\frac{r^2}{2} - \frac{r^4}{4}\right)\Big]_0^3$$

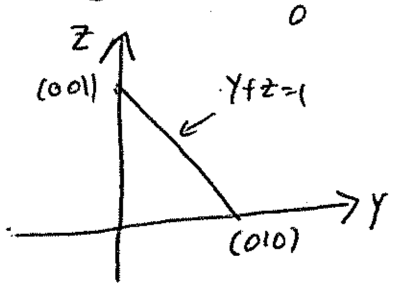
$$= 4\pi\left(\frac{81}{2} - \frac{81}{4}\right) = 4\pi \frac{81}{4} = 81\pi$$

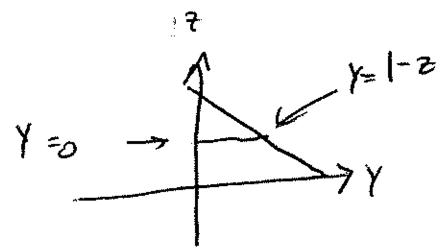
(5)



The front face is $x+2z=2$ (because $(0,0,1)$, $(2,0,0)$ and $(2,1,0)$ all satisfy this equation.) So the integral is

$$\iint \int_0^{2-2z} f(x,y,z)\,dx\,\underbrace{dy\,dz}_{\text{in some order}} = \boxed{\int_0^1 \int_0^{1-z} \int_0^{2-2z} f(x,y,z)\,dx\,dy\,dz}$$





S. MR 33

Hall 38

Lloyd 8

TT 48

Jones 30