1 We must show that $\vec{F}(\vec{c}(t)) = \vec{c}\,'(t)$ for all t.

LHS $= \vec{F}(\vec{c}(t)) = \vec{F}(\sin t, \cos t, e^t) = (\cos t, -\sin t, e^t)$

RHS $= (\cos t, -\sin t, e^t)$

LHS = RHS

so $\vec{c}(t)$ is a flow line of $\vec{F}$.

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2 $\vec{\nabla} \cdot \vec{V} = 1 + 1 + 1 = 3$

3 $\text{Curl}\,\vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = \vec{i}(x - x) - \vec{j}(y - y) + \vec{k}(z - z) = 0$

4 $\vec{c}\,'(t) = (3t^2, -e^{-t}, -\frac{\pi}{2}\sin(\frac{\pi t}{2}))$

$\vec{c}\,'(1) = (3, -e^{-1}, -\frac{\pi}{2})$

$\vec{c}(1) = (2, e^{-1}, 0)$

tangent line is $\dfrac{x-2}{3} = \dfrac{y-e^{-1}}{-e^{-1}} = \dfrac{z-0}{-\frac{\pi}{2}}$

5 Let $t = x$ so $y = x^{\frac{2}{3}} = t^{\frac{2}{3}}$ and $z = x^{\frac{2}{5}} = t^{\frac{2}{5}}$

arc length $= \displaystyle\int_1^4 \sqrt{1^2 + \left(\tfrac{2}{3}t^{-\frac{1}{3}}\right)^2 + \left(\tfrac{2}{5}t^{-\frac{3}{5}}\right)^2}\, dt$

6 $\displaystyle\int_0^1 xy + \frac{y^2}{2}\Big]_1^{e^x} = \int_0^1 \left(xe^x + \frac{e^{2x}}{2} - x - \frac{1}{2}\right) dx = \left(xe^x - e^x + \frac{e^{2x}}{4} - \frac{x^2}{2} - \frac{1}{2}x\right)\Big]_0^1 = \left(e - e + \frac{e^2}{4} - \frac{1}{2} - \frac{1}{2}\right) - \left(0 - 1 + \frac{1}{4}\right)$

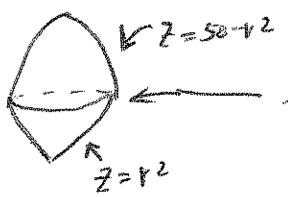
$\int xe^x = xe^x - \int e^x dx = xe^x - e^x$

$u = x \quad v = e^x$

$du = dx \quad dv = e^x dx$

$= \dfrac{e^2}{4}$

7



the intersection is $z = 25$, $r = 5$

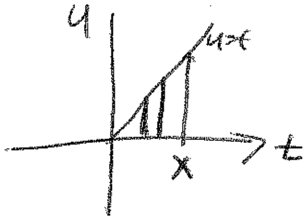
$\text{Vol} = \displaystyle\int_0^{2\pi}\int_0^5\int_{r^2}^{50-r^2} r\, dz\, dr\, d\theta = 2\pi \int_0^5 (50 - 2r^2) r\, dr$

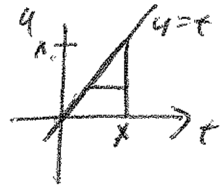
$= 2\pi \; \dfrac{50r^2}{2} - \dfrac{2r^4}{4}\Big]_0^5$

$= 2\pi\left(\dfrac{50(25)}{2} - \dfrac{2(25)^2}{4}\right) = 2\pi\left(625 - \dfrac{625}{2}\right)$

$= 625\pi$

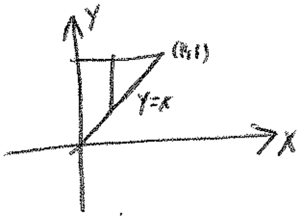
(8) $\displaystyle\int_0^x\int_0^t F(u)\,du\,dt = \int_0^x\int_u^x F(u)\,dt\,du = \int_0^x F(u)\,t\Big]_u^x\,du$

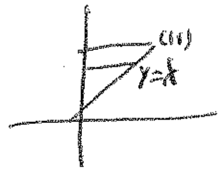




$\displaystyle = \int_0^x F(u)(x-u)\,du.$

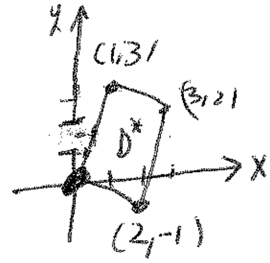
(9)





integral $\displaystyle = \int_0^1\int_0^y e^{y^2}\,dx\,dy$

$\displaystyle = \int_0^1 y e^{y^2}\,dy = \frac{1}{2}e^{y^2}\Big]_0^1 = \frac{1}{2}(e-1)$



(10) Multiplication by $\begin{bmatrix}2 & 1\\ -1 & 3\end{bmatrix}$ carries D to D^*

So Multiplication by $\frac{1}{7}\begin{bmatrix}3 & -1\\ 1 & 2\end{bmatrix}$ carries D^* to D

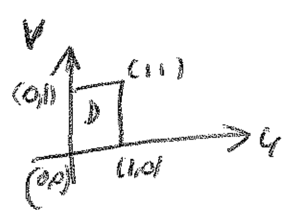
Define $T(x,y) = \left(\frac{3}{7}x - \frac{1}{7}y,\ \frac{1}{7}x + \frac{2}{7}y\right)$

check $T(0,0) = (0,0)$

$T(2,-1) = (1,0)$

$T(3,2) = (1,1)$

$T(1,3) = (0,$ [redacted] $)$



We see that T carries D^* to the square