1999 Exam 2

① $\vec N = \overrightarrow{(111)(223)} \times \overrightarrow{(111)(134)} = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & 1 & 2 \\ 0 & 2 & 3 \end{vmatrix}$

$= -\vec i - 3\vec j + 2\vec k$

Plane is $-(x-1) - 3(y-1) + 2(z-1) = 0$

$-x - 3y + 2z = -2$

(1,1,1) works ✓
(2,2,3) works ✓
(1,3,4) works ✓

② Gradients are ⊥ to level sets. we look at the level set $0 = \underbrace{x^2+y^2-z}_{f}$. $\vec N = \vec\nabla f\big|_{(1,2)} = 2x\vec i + 2y\vec j - \vec k\big|_{(1,2)}$

$= 2\vec i + 4\vec j - \vec k$

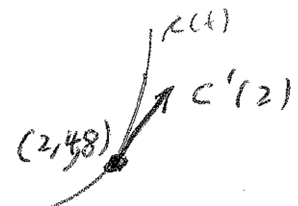
The plane through $(1,2,5)$ ⊥ $\vec N$ is $2(x-1) + 4(y-2) - (z-5) = 0$

Note the z-coordinate is 5

③ $c'(t) = (1, 2t, 3t^2)$

$c'(2) = (1, 4, 12)$

The line through $(2,4,8)$ ∥ to $c'(2)$ is $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-8}{12}$



4) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim\limits_{(x,y)\to(0,0)} \frac{0}{0+y^2} = \lim\limits_{y\to0} 0 = 0$

along $x=0$

$\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim\limits_{x\to0} \frac{x^2}{2x^2} = \lim\limits_{x\to0} \frac{1}{2} = \frac{1}{2}$ different.

along $y=x$

so $\lim_{(x,y)\to(0,0)} \frac{x\,y}{x^2+y^2}$ Does Not exist

5) Speed is constant. But speed $= \|\vec{c}\,'(t)\| = \sqrt{\vec{c}\,'(t)\cdot\vec{c}\,'(t)}$

Thus $\vec{c}\,'(t)\cdot\vec{c}\,'(t) =$ constant

Take $\frac{d}{dt}$ of both sides $\vec{c}\,'(t)\cdot\vec{c}\,''(t) + \vec{c}\,''(t)\cdot\vec{c}\,'(t) = 0$

Thus $2\,\vec{V}(t)\cdot\vec{a}(t) = 0$

Thus $\vec{V}(t)\cdot\vec{a}(t) = 0$

Thus $\vec{V}(t) \perp \vec{a}(t)$

6) $AL = \int_1^2 \|\vec{c}\,'(t)\|\,dt = \int_1^2 \|(2, 2t, \tfrac{1}{t})\|\,dt = \int_1^2 \sqrt{4+4t^2+\tfrac{1}{t^2}}\,dt$

$= \int_1^2 \sqrt{(2t+\tfrac{1}{t})^2}\,dt = \int_1^2 (2t+\tfrac{1}{t})\,dt$

$= t^2 + \ln t\,\Big]_1^2 = 4 + \ln 2 - 1$

7) Curvature $= \frac{\|c'(t)\times c''(t)\|}{\|c'(t)\|^3} = \frac{\|c''(t)\|}{2} = \frac{\|(-\cos t, -\sin t, 0)\|}{2} = \frac{1}{2}$

$\|c'(t)\| = \|(-\sin t, \cos t, 1)\| = \sqrt{1+1} = \sqrt{2}$

Speed is constant so (see #5)

$c' \perp c''$

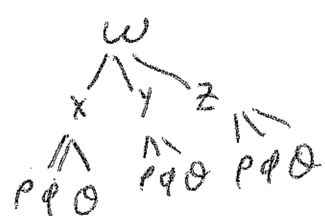
So $\|c'\times c''\| = \|c'\|\,\|c''\|$

8) 

$x = r\cos\theta = \rho\sin\varphi\cos\theta$

$y = r\sin\theta = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

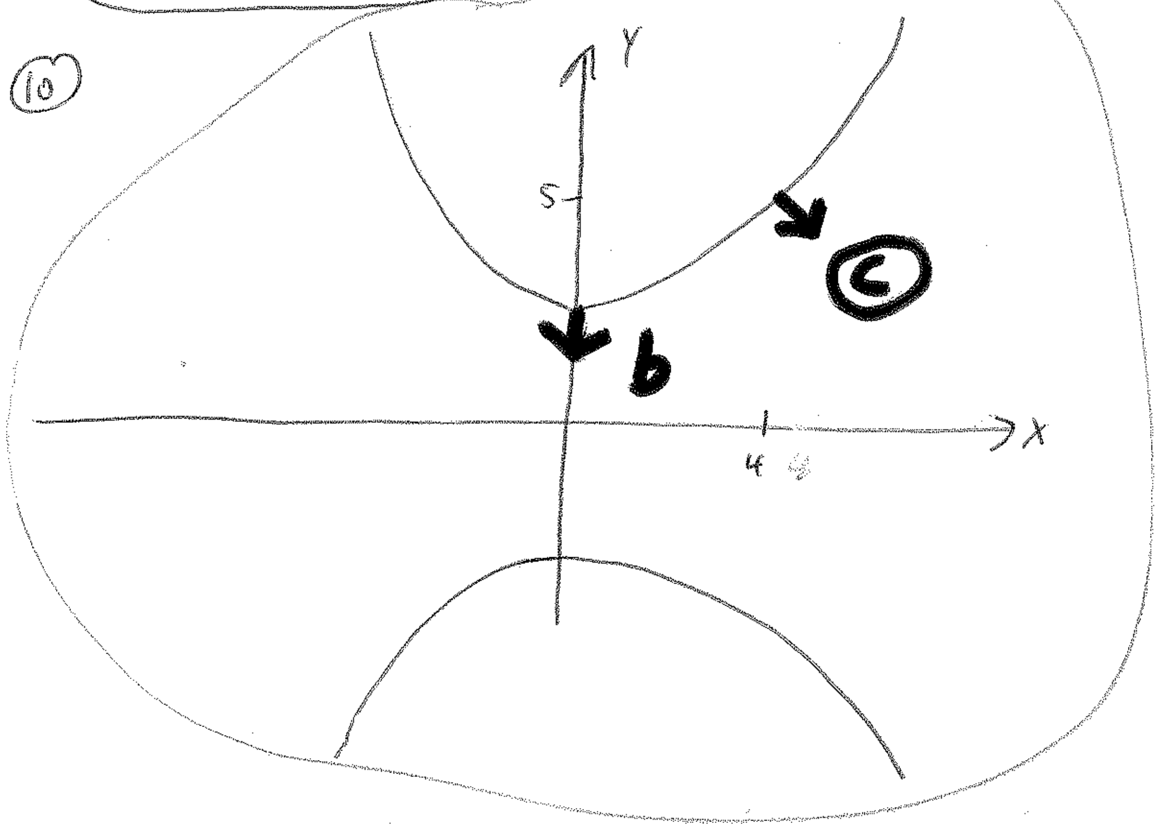


$\frac{\partial w}{\partial\varphi} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial\varphi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial\varphi} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial\varphi}$

$\frac{\partial w}{\partial\varphi} = \frac{\partial w}{\partial x}\rho\cos\varphi\cos\theta + \frac{\partial w}{\partial y}\rho\cos\varphi\sin\theta + \frac{\partial w}{\partial z}(-\rho\sin\varphi)$

(9) $x(t) = 2\cos t$
$y(t) = 3\sin t$

(10)



(b) $\vec{\nabla} f = -2x\vec{i} + 2y\vec{j}$

$\vec{\nabla} f|_{(0,3)} = 6\vec{j}$ $\quad -\frac{1}{10}\vec{\nabla} f|_{(0,3)} = -.6\vec{j}$

(c) $\vec{\nabla} f|_{(4,5)} = -8\vec{i} + 10\vec{j}$ $\quad -\frac{1}{10}\vec{\nabla} f|_{(4,5)} = .8\vec{i} - \vec{j}$

Of course, gradients are perpendicular to level sets.