① Let $\vec{N} = \overrightarrow{(0,0,0)\,(1,1,1)} \times \overrightarrow{(0,0,0)\,(2,3,4)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

$= \vec{i} - \vec{j}(2) + \vec{k}$

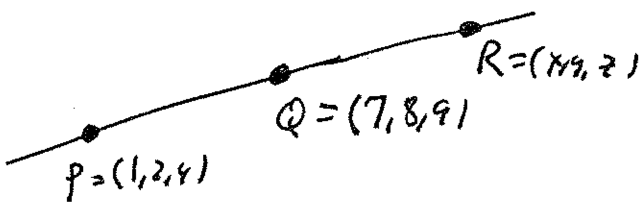
$\boxed{\text{1999 Exam 1}}$

The plane through (0,0,0) which is $\perp$ to $\vec{N}$ is

$x - 2y + z = 0$

<u>check</u> (0,0,0) works ✓
(1 1 1) works ✓
(2,3,4) works ✓

②



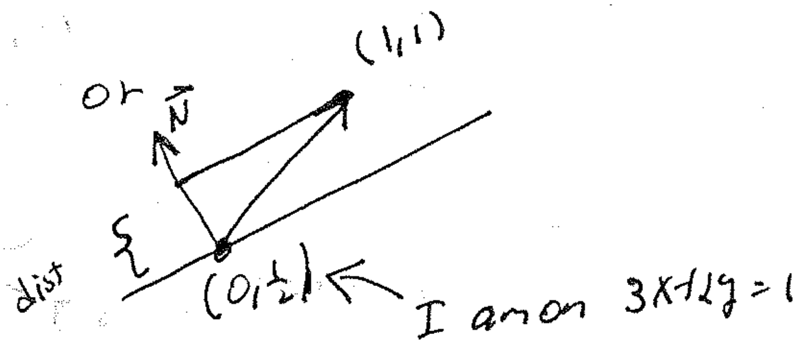
$\overrightarrow{PR} = t\,\overrightarrow{PQ}$

$$\begin{cases} x - 1 = t\,6 \\ y - 2 = t\,6 \\ z - 4 = t\,5 \end{cases}$$

check $t=0$ gives (1, 2, 4)
$t=1$ gives (7, 8, 9)

③ use pg 76 #16 dist $= \dfrac{|3+2-1|}{\sqrt{9^2+4}} = \dfrac{4}{\sqrt{13}}$

or



$$\text{dist} = \left\| \operatorname{proj}_{\vec N} \overrightarrow{(0,\tfrac12)(1,1)} \right\| = \left\| \frac{\vec N \cdot (\vec i + \tfrac12 \vec j)}{\|\vec N\|^2} \vec N \right\|$$

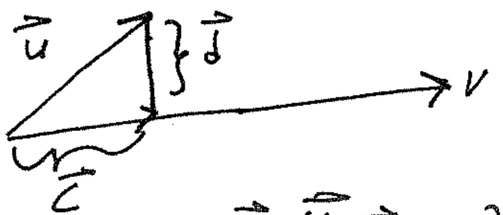
(c

$\vec N = 3\vec i + 2\vec j$

because gradients are $\perp$ level sets

$$= \frac{3+1}{\sqrt{13}} = \frac{4}{\sqrt{13}}$$

(4)



$$\vec c = \operatorname{proj}_{\vec v} \vec u = \frac{\vec v \cdot \vec u}{\vec v \cdot \vec v}\vec v = \frac{2+6}{1+4+9}\vec v = \frac{8}{14}\vec v = \frac{4}{7}(\vec i + 2\vec j + 3\vec k)$$

$\uparrow$ $\vec c$

$$\vec d = \vec u - \vec c = 2\vec i + 3\vec j - \left(\tfrac47 \vec i + \tfrac87 \vec j + \tfrac{12}{7}\vec k\right)$$

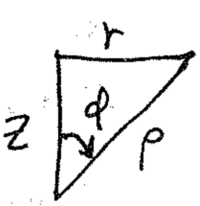
$$= \frac{10}{7}\vec i + \frac{13}{7}\vec j - \frac{12}{7}\vec k = \vec d$$

$\vec c + \vec d = \vec u$ ✓

$\vec c$ is a multiple of $\vec v$ ✓

$\vec d \perp \vec v$ because $10 + 26 - 36 = 0$ ✓

(5)



$z = \rho\cos\varphi$

$r = \rho\sin\varphi$

$x = r\cos\theta = \rho\sin\varphi\cos\theta$

$y = r\sin\theta = \rho\sin\varphi\sin\theta$

$z = x^2 - y^2$ is $\rho\cos\varphi = \rho^2\sin^2\varphi\,(\cos^2\theta - \sin^2\theta)$

i.e. $\cos\varphi = \rho\sin^2\varphi\cos 2\theta$

(6) The level set for $f(x,y) = c$ is

$$c = \sqrt{100 - x^2 - y^2}$$

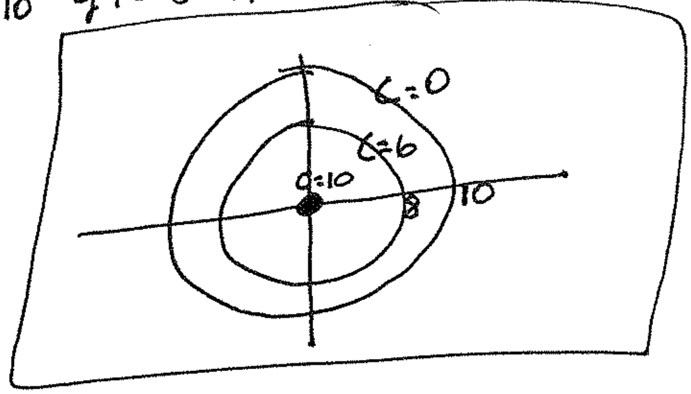
$$c^2 = 100 - x^2 - y^2$$

$$x^2 + y^2 = 100 - c^2$$

so level $c=0$ gives circle of radius 10 center $(0,0)$

level $c=6$ gives circle of radius 8 center $(0,0)$

level $c=10$ gives circle of radius 0 center $(0,0)$



(7) $$\lim_{x\to 0} \frac{\cos x - 1}{x^2} = \lim_{x\to 0} \frac{-\sin x}{2x} = \lim_{x\to 0} \frac{-\cos x}{2} = -\frac{1}{2}$$

L'hopital

(8) The limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{y\to 0} \frac{0}{y^2} = \lim_{y\to 0} 0 = 0$$

along $x=0$

diffe

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \lim_{x\to 0} \frac{1}{2} = \frac{1}{2}$$

along $x=y$

If the limit were to exist then every approach to $(0,0)$ would give the same answer. So this limit does not exist.

(9) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2+2} = \frac{0}{2} = 0$

(10) The line is $x = 3t-2$
$y = 4t+3$
$z = t-1$

The line hits the plane when

$(3t-2) - 2(4t+3) + 3(t-1) + 7 = 0$

$3t - 8t + 3t - 2 - 6 - 3 + 7 = 0$

$-2t - 4 = 0$

$-4 = 2t$

$-2 = t$

The intersection point is $x = -8$
$y = -5$
$z = -3$

check

$\frac{-8+2}{3} = \frac{-5-3}{4} = -3+1$ ✓

and

$-8 - 2(-5) + 3(-3) + 7 = -8 + 10 - 9 + 7 = 0$ ✓